

TOPOLOGY OPTIMIZATION OF 2D STRUCTURES FOR COMPLIANCE AND FUNDAMENTAL FREQUENCY OBJECTIVES USING MATLAB

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Abstract

This paper presents a unified and fully MATLAB-based framework for the topology optimization of two-dimensional (2D) structures, addressing both static and dynamic performance objectives. The method combines a custom finite element code for plane stress analysis with a density-based SIMP approach and a gradient-based optimizer. In the static formulation, the goal is to minimize structural compliance under a volume constraint, producing stiff and material-efficient designs. The dynamic extension focuses on maximizing the fundamental eigenfrequency (or equivalently, minimizing the first eigenperiod), which is essential for vibration-sensitive applications. The same finite element framework is used to compute stiffness and mass matrices, enabling consistent implementation of both optimization objectives. To improve physical realism in dynamic problems, lumped masses can be assigned to specific nodes, guiding material distribution in otherwise under-constrained regions. Numerical examples, including a cantilever beam under static loading and a vibration-optimized beam with added mass, demonstrate the effectiveness of the framework. The dynamic results are in very good agreement with published literature, validating the proposed approach. Overall, the framework offers a flexible, transparent, and extensible platform for topology optimization research and education.

Keywords: Topology Optimization, SIMP Method, Compliance Minimization, Eigenfrequency Optimization, Gradient-Based Optimization, MATLAB.

1 INTRODUCTION

Topology optimization has emerged as a powerful tool in structural engineering for designing efficient material layouts that meet performance objectives while minimizing material usage. With increasing demands for sustainable and lightweight structures, especially in civil and structural applications [1], topology optimization offers a systematic approach to identifying optimal structural forms that satisfy given constraints [2]. Its potential for reducing self-weight, improving load-bearing efficiency, and enabling innovative designs makes it especially attractive in the context of performance-based engineering and modern computational methods.

In the domain of structural design, compliance minimization has been one of the most widely used objective functions in topology optimization. Minimizing compliance effectively maximizes stiffness under prescribed loads and boundary conditions, leading to material-efficient designs that are mechanically optimal [3]. This static objective is particularly relevant in applications where structural rigidity and load resistance are critical [4]. However, focusing solely on static performance can overlook critical vulnerabilities when structures are subjected to dynamic excitations—such as seismic events [5] or operational vibrations common in earthquake-prone regions or sensitive environments—which necessitate consideration of dynamic objectives like stiffness and energy dissipation [6].

In this regard, frequency-based topology optimization has gained attention as a complementary strategy. Instead of focusing on external loads, this approach considers the dynamic characteristics of the structure, specifically its natural frequencies or vibration modes [7]. Maximizing the fundamental eigenfrequency, or equivalently minimizing the first eigenperiod, results in configurations that are inherently stiffer and more resistant to dynamic excitation [8]. This objective is especially meaningful in the context of earthquake engineering, where a structure's fundamental period plays a key role in determining its dynamic amplification and resonance behavior [9].

In recent years, the importance of topology optimization has been significantly reinforced by rapid advances in artificial intelligence (AI) [10-12], generative design, and additive manufacturing. AI-driven approaches—particularly those based on machine learning and deep learning—have enabled more efficient exploration of large and complex design spaces, facilitating faster convergence and improved performance in structural optimization tasks [13]. Moreover, the integration of AI into civil and structural engineering workflows is reshaping both research and education, offering new possibilities for automating design, enhancing resilience, and supporting decision-making [14, 15]. Generative models, such as generative adversarial networks (GANs) [16], can produce innovative and non-intuitive structural configurations that complement conventional topology optimization methods by proposing high-performing, unconventional layouts. Simultaneously, additive manufacturing technologies have progressed to a level where the intricate geometries generated by these optimization processes can now be fabricated with high accuracy and minimal material waste [17]. This convergence of AI-based design automation and manufacturing capabilities has redefined the practical impact of topology optimization, positioning it at the core of modern engineering workflows and enabling structures that are not only optimized but also buildable. These trends underscore the importance of developing versatile optimization frameworks—such as the one proposed in this paper—that can accommodate evolving design paradigms and support future engineering innovations.

This paper presents a unified MATLAB-based framework for topology optimization of two-dimensional structures, addressing both static and dynamic performance criteria. The framework combines a custom finite element implementation for plane stress elements with a density-based Solid Isotropic Material with Penalization (SIMP) approach and a gradient-based optimization algorithm. For static analysis, the objective is to minimize compliance of the structure

under a volume constraint. The framework is then extended to solve dynamic problems by incorporating the mass matrix and solving the generalized eigenvalue problem. The dynamic formulation aims to maximize the fundamental frequency (or equivalently minimize the first eigenperiod) without taking into account any external loads, thus exploring a pure vibration-based optimization objective. Numerical examples for both cases are presented and discussed, with the goal of highlighting the similarities, differences, and potential synergies of integrating static and dynamic criteria in topology optimization.

2 BACKGROUND AND LITERATURE REVIEW

Structural topology optimization is a computational methodology that seeks the optimal material distribution within a given domain, subject to specified objectives and constraints. Unlike traditional size or shape optimization, topology optimization allows the introduction of new holes and connectivity changes, leading to innovative and non-intuitive structural designs [2, 13]. The method gained traction in the late 20th century, building upon earlier concepts such as Michell's truss theory from the early 1900s [18].

A key breakthrough came with the work of Bendsøe and Kikuchi [19], who introduced a homogenization-based method that resolved the ill-posedness of classical 0-1 formulations by allowing continuous material densities between 0 (void) and 1 (solid). This relaxation enabled the application of gradient-based optimization algorithms and paved the way for systematic topology optimization.

Among the various approaches developed, the density-based SIMP method has become the most widely adopted. Originally proposed by Bendsøe [3] and later formalized by Zhou and Rozvany [4], SIMP offers a simple material interpolation model and has become the de facto standard due to its efficiency, ease of implementation, and wide applicability [20, 21]. SIMP's effectiveness is further supported by filtering strategies that suppress numerical instabilities like checkerboarding [22].

In the SIMP framework, the design domain is discretized into finite elements, each associated with a density variable. These densities influence the stiffness and other properties of the material through penalized interpolation schemes. The result is a continuous optimization problem that is computationally tractable yet leads to nearly binary designs. The method has been successfully extended to address multiple objectives, constraints, and physical phenomena [23].

Minimizing compliance—equivalent to maximizing stiffness for a given volume—has been the classical objective in structural topology optimization. Early continuum formulations appeared in the 1960s and 70s through the works of Sheu and Schmit [24], Rozvany [25], and others, but they suffered from non-existence of solutions in discrete form [26]. The introduction of continuous density methods like homogenization and SIMP addressed this issue and sparked a wave of research throughout the 1990s and 2000s [3, 4, 27, 28].

Building on these developments, Rozvany provided theoretical solutions for benchmark cases, and Bendsøe and Sigmund's comprehensive book summarized key advances up to the early 2000s [20]. Sigmund's 99-line MATLAB code made the method accessible to a wide audience [27], and was later extended to 3D by Andreassen et al. [29]. Today, compliance minimization remains a benchmark problem for testing new algorithms, including level-set methods, phase-field models, and various optimization strategies.

In contrast, eigenfrequency optimization—focused on tuning the natural vibration characteristics of structures—has been comparatively less studied. Pioneering work in this area began with Díaz and Kikuchi [30], who used the homogenization method to maximize the fundamental frequency of 2D structures. Subsequent contributions extended this approach using SIMP and evolutionary methods [7, 8, 31-33]. These studies demonstrated that vibrational

performance can be significantly improved by optimizing the material layout, though the problem is inherently more complex than compliance minimization.

A major challenge in frequency optimization is the handling of multiple or closely spaced eigenvalues, which can complicate sensitivity analysis. Seyranian et al. [34] addressed these issues using perturbation methods to ensure differentiability of eigenvalues. Other works explored bandgap creation, prescribed mode shapes, and dynamic compliance objectives, expanding the scope of vibration-related optimization [7, 35].

Among non-gradient approaches, the Evolutionary Structural Optimization (ESO) method introduced by Xie and Steven [36] and its extension Bi-directional ESO (BESO) by Querin et al. [37] offer an intuitive alternative to SIMP. These algorithms iteratively remove (ESO) or both remove and add (BESO) material based on performance indicators. Both static and dynamic applications have been demonstrated—for example, Huang et al. [32] applied ESO to maximize natural frequencies, and Zuo et al. [33] developed BESO for frequency optimization. However, these evolutionary approaches do not guarantee global optimality and often rely on heuristic parameters that can affect convergence. In contrast, SIMP-based methods rest on stronger theoretical grounds and integrate effectively with advanced solvers like MMA and SQP [21, 38].

Despite the inherent conceptual and computational challenges, eigenfrequency optimization remains an important area of study. When properly formulated using SIMP and enhanced with techniques such as filtering, penalization, and mode tracking, it can yield physically meaningful, high-performance designs. This study focuses on SIMP-based topology optimization for both compliance and frequency objectives, using in-house MATLAB codes tailored for 2D structural problems. While the present work is limited to planar structures for clarity and efficiency, the same methods are readily extendable to 3D.

3 METHODOLOGY

3.1 Overview of the Topology Optimization Framework

All analyses presented in this work have been carried out using a custom-developed MATLAB framework created entirely in-house. The purpose of this framework is to provide a unified, transparent environment for structural topology optimization under both static and dynamic objectives. The code has been developed in modular steps, each building upon the previous, and integrates finite element analysis, topology optimization algorithms, and eigenvalue computations. The development can be described as a four-step process:

1. Static Analysis Module: The first step involved creating a finite element code for plane stress analysis. This module serves as the computational backbone of the framework, solving the static response of 2D structures. The domain is discretized using 4-noded quadrilateral elements. Users can select between reduced integration (1 Gauss point per element) and full integration (4 Gauss points per element). The code computes nodal displacements, element stresses, and internal forces. A basic mesh generator is included to create structured meshes for rectangular geometries, while unstructured or complex meshes generated in external software such as Abaqus can also be imported via formatted text files.

2. Topology Optimization Module (Static Case): The second module implements the topology optimization procedure for static load cases. It builds directly upon the static analysis code. The optimization objective is to minimize the compliance of the structure—effectively maximizing stiffness—subject to a constraint on the allowable volume fraction of material. The SIMP method is employed, where material densities are treated as continuous design variables. The stiffness matrix is updated based on the density distribution at each iteration. A

mathematical programming approach is used, specifically MATLAB's `fmincon` function with the Sequential Quadratic Programming (SQP) algorithm, to perform the optimization.

3. Dynamic Analysis Module: The third step involved the development of a dynamic analysis module that calculates the natural frequencies, mode shapes (eigenmodes), and eigenperiods of 2D structures. This module builds upon the same finite element formulation used in the static case. Each element's mass is computed based on its current material density and geometry, and the resulting mass is consistently distributed to the nodes to assemble the global mass matrix. The generalized eigenvalue problem is then solved to determine the dynamic properties of the structure. Damping effects are not considered in this analysis. In addition to the distributed element-based mass, the framework allows users to introduce lumped masses at specific nodes. Users can activate this option and freely specify the nodes where these masses should be applied, along with the corresponding mass values. This feature enables the simulation of added mass effects—such as equipment or concentrated loads—at user-defined locations, providing greater flexibility in modeling real-world dynamic scenarios and fine-tuning the structural response.

4. Topology Optimization Module (Dynamic Case): Finally, the dynamic analysis module was integrated with the optimization engine to enable topology optimization under vibration-related objectives. Unlike the static case, this dynamic formulation does not include external loads. Instead, the objective is to minimize the first eigenperiod (or equivalently, to maximize the first natural frequency) under volume fraction constraints. In each iteration, the current material distribution defines the stiffness and mass matrices, which are used to solve the eigenvalue problem. The optimization proceeds by updating the material layout to achieve the shortest possible fundamental period, effectively enhancing the dynamic stiffness of the structure.

This four-stage modular approach ensures that the framework remains flexible, extendable, and suitable for further developments. All modules are tightly integrated in MATLAB, allowing consistent use of the same mesh and data structures throughout static and dynamic analyses. In all cases, the optimization is carried out using MATLAB's `fmincon` function with the Sequential Quadratic Programming (SQP) algorithm.

3.2 Static Problem: SIMP-based Compliance Minimization

In the static topology optimization problem, the goal is to determine the optimal distribution of material within a fixed design domain such that the structure is as stiff as possible, under given loads and boundary conditions. This is typically formulated as the minimization of the total compliance of the structure, which is equivalent to minimizing the work done by external forces — or, in other words, maximizing stiffness.

In SIMP-based topology optimization, each finite element e is assigned a density variable ρ in $[0,1]$, where $\rho_e=1$ represents solid material and $\rho_e=0$ represents a void. The elastic stiffness of each element is modeled as a function of its density, typically

$$E_e(\rho_e) = \rho_e^p E_0, \quad (1)$$

where E_0 is the Young's modulus of the solid material and p is a penalization factor (commonly $p=3$) [39]. The role of this power-law penalization is to push intermediate densities toward 0 or 1 during the optimization – higher values of p penalize partial material and encourage a nearly binary (black-and-white) solution. Similarly, other properties like mass density can be interpolated, usually linearly as ρ_e times the solid mass density for each element. This will be discussed in detail in the dynamic problem section. By using a continuous design variable

and interpolation, SIMP enables the use of gradient-based optimizers while approximating the discrete nature of the material distribution problem.

The standard formulation of the compliance minimization problem using the SIMP method is:

$$\begin{aligned} \min_{\rho_e \in [\rho_{\min}, 1]} \quad & C(\boldsymbol{\rho}) = \mathbf{U}^T \mathbf{F} \\ \text{subject to:} \quad & \\ \sum_{e=1}^{N_e} \rho_e v_e \leq V_f \cdot V_0 \quad & , \\ \mathbf{K}(\boldsymbol{\rho}) \mathbf{U} = \mathbf{F} \end{aligned} \quad (2)$$

where:

- $\rho_e \in [\rho_{\min}, 1]$ is the design variable representing the relative density of element e ,
- $\mathbf{K}(\boldsymbol{\rho})$ is the global stiffness matrix, assembled from the contributions of all elements. For each element e , the element stiffness matrix is defined as $\mathbf{K}_e = \rho_e^p \cdot \mathbf{K}_e^0$,
- $\mathbf{K}_e^0$ is the stiffness matrix of the element assuming full solid material.
- p is the penalization power (commonly chosen as 3),
- $\mathbf{U}$ is the global displacement vector,
- N_e is the total number of elements,
- v_e is the volume of element e .
- $V_f \in (0, 1]$ is the volume fraction, e.g., $V_f = 0.4$ for 40%.
- $V_0 = \sum_{e=1}^{N_e} v_e$ is the total volume of the design domain (i.e., all elements at full density).
- $\rho_{\min}$ is a small positive number to prevent singularities in the stiffness matrix,

It should be noted that in SIMP-based compliance optimization, the objective can be also expressed equivalently as minimizing total strain energy, $1/2 \cdot \mathbf{U}^T \mathbf{F}$, which is half the compliance [39]. To avoid numerical instabilities such as checkerboarding and mesh dependency, a density filtering technique is applied to the design variables. In this work, we adopted the two-field SIMP approach. The use of a separate filtered density field ($\tilde{\rho}$) helps control the minimum feature size of the topology and ensures mesh independence, as well as improved manufacturability of the final design. The filtering is performed using a spatially weighted average of neighboring design variables, computed as follows [40]:

$$\tilde{\rho}_e = \frac{\sum_{i \in \mathcal{N}_e} w_{ei} \rho_i}{\sum_{i \in \mathcal{N}_e} w_{ei}}, \quad (3)$$

where:

- $\mathcal{N}_e$ is the set of elements within a specified filter radius $r_{\min}$ around element e ,
- $w_{ei} = r_{\min} - \text{dist}(e, i)$ is the linear weighting function based on the distance between elements e and i .

This linear decay ensures that elements closer to e have a stronger influence on its filtered density $\tilde{\rho}_e$, while elements farther than the radius $r_{\min}$ have no influence at all. The filtered densities $\tilde{\rho}_e$ are then used in the material interpolation scheme for computing the element stiffness values via the SIMP relation:

$$E_e(\tilde{\rho}_e) = \tilde{\rho}_e^p E_0 \quad (4)$$

By using this two-field formulation, the optimizer operates on the smooth design variable field ρ , while the filtered field $\tilde{\rho}$ determines the actual stiffness distribution used in the analysis. This strategy is effective in eliminating numerical artifacts and contributes to the robustness and convergence of the optimization process. At each iteration, the compliance is evaluated based on the current filtered density distribution, and the sensitivity of the objective function with respect to the design variables is computed using the adjoint method.

3.3 Dynamic Problem: SIMP-based Fundamental Period Minimization

While compliance minimization is a static objective, the SIMP framework is flexible and has been applied to other objectives as well. In our case, in the dynamic formulation of topology optimization, the goal shifts from minimizing static compliance to improving the structure's dynamic performance, specifically its natural frequency characteristics. The most common approach is to maximize the fundamental eigenfrequency ω_1 , or equivalently, minimize the first eigenperiod $T_1 = 2\pi/\omega_1$. Both formulations are mathematically equivalent since increasing ω_1 reduces T_1 , which reflects a stiffer and dynamically more robust structure. The formulation of the dynamic problem is as follows:

$$\begin{aligned} \min_{\rho_e \in [\rho_{\min}, 1]} T_1 = \frac{2\pi}{\omega_1(\rho)} \quad \text{or} \quad \max_{\rho_e \in [\rho_{\min}, 1]} \omega_1(\rho) \\ \text{subject to:} \\ \sum_{e=1}^{N_e} \rho_e v_e \leq V_f \cdot V_0 \\ \mathbf{K}(\rho)\mathbf{u} = \lambda \mathbf{M}(\rho)\mathbf{u} \end{aligned} \quad (5)$$

where:

- $\mathbf{M}(\rho)$ is the global mass matrix, a diagonal matrix assembled from the contributions of all elements. For each element e , the element mass matrix is defined as $\mathbf{M}_e = \rho_e^q \cdot \mathbf{M}_e^0$,
- $\mathbf{M}_e^0$ is the consistent mass matrix of the element assuming full solid material.
- q controls the penalization of the mass relative to the density field and is typically set to $q=1$ [41],
- $\lambda = \omega^2$ is the eigenvalue corresponding to the square of the natural frequency,
- $\mathbf{u}$ is the mode shape (eigenvector) of the corresponding frequency,
- ω_1 is the first natural frequency,
- T_1 is the first eigenperiod.

To solve the generalized eigenvalue problem described in Eq. (5), one must find the values of λ that satisfy the condition:

$$\det(\mathbf{K}(\rho) - \lambda \mathbf{M}(\rho)) = 0 \quad (6)$$

This condition ensures non-trivial solutions for the eigenvector $\mathbf{u}$. The resulting eigenvalues λ correspond to the squared natural frequencies of the system, i.e., $\lambda = \omega^2$. In our implementation, the lowest eigenvalue—associated with the first natural frequency—is extracted numerically using MATLAB's `eigs` function. Specifically, we use the command: `eigs(K, M, 1, 'SM')`, which computes the smallest-magnitude eigenvalue of the matrix pair $(\mathbf{K}, \mathbf{M})$. This value is then used to compute the fundamental frequency $\omega_1 = \sqrt{\lambda_1}$, which is subsequently used in the topology optimization objective.

This dynamic optimization problem introduces unique challenges that differ fundamentally from the static compliance formulation. Since the objective function is no longer driven by externally applied forces, the optimizer seeks material distributions that maximize global stiffness primarily with respect to the dominant vibration modes. This can result in material accumulating in peripheral or isolated regions of the structure, potentially yielding non-functional or impractical designs. To address this, design regularization techniques are necessary to guide the optimization towards manufacturable and mechanically sound solutions. These techniques include density filtering, imposition of a minimum length scale, and penalization of intermediate density values.

Another effective strategy, as proposed in [41, 42], involves the introduction of a lumped mass at a predefined location within the structure. This point is typically selected to correspond with the region where an external load would be expected in a static analysis, thereby emulating practical constraints and encouraging material allocation around structurally relevant areas. This is the approach we have implemented in our framework, and its effects on the resulting topologies will be discussed in the numerical results section.

4 NUMERICAL EXAMPLES

4.1 Cantilever Beam A: Static Analysis Case

The first numerical example considers a cantilever beam subjected to a static load. The beam is fixed along its left vertical edge, while a concentrated downward point load is applied at the midpoint of the right edge. The geometry of the structure is illustrated in Figure 1, where the support on the left is a fixed support (i.e. all nodes are pinned) and the force on the right is $F = 100$ N (pointing downwards). In this example, all quantities are expressed using a consistent unit system: millimeters (mm) for distance, newtons (N) for force, megagrams (Mg) for mass, and seconds (s) for time. These units ensure dimensional consistency throughout the analysis.

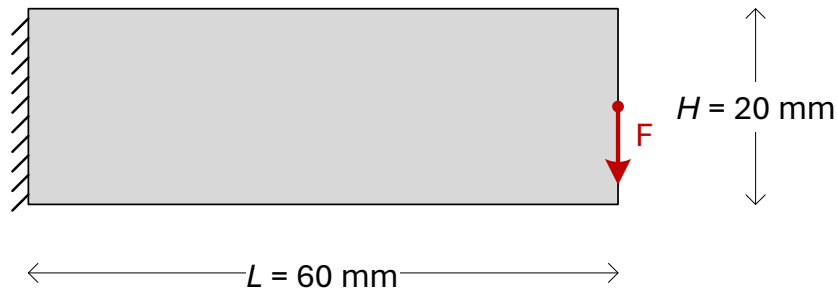


Figure 1. Cantilever beam A used in the static case example [43].

Material and other properties are given below:

- Density: $\rho = 7.85 \text{ Mg/m}^3 = 7.85 \times 10^{-9} \text{ Mg/mm}^3$,
- Young's modulus: $E = 200 \text{ GPa} = 200,000 \text{ MPa}$,

- Poisson's ratio: $\nu = 0.3$,
- Thickness: $t = 5$ mm.

The mesh used in this study consists of 46×16 elements, resulting in a total of 736 elements—and therefore 736 design variables for the topology optimization problem. The discretization yields 799 nodes, leading to 1,598 total degrees of freedom (DOFs). With 17 nodes fully constrained along the left edge, the number of active (free) DOFs in the analysis is reduced to 1,564. The topology optimization settings are the following:

- Volume fraction: $V_f = 0.4$,
- SIMP penalty for stiffness matrix: $p = 3$,
- Filter radius: $R = 2.5$ (mm),
- Initial material distribution: Uniform, with $x_0 = 0.5$ (Initial volume fraction is 0.5 for the starting design).

Figure 2 illustrates the evolution of the material layout during the topology optimization process for the cantilever beam example. The progression is shown over 70 iterations, with snapshots taken every 5 steps. At iteration 0, the material is uniformly distributed across the entire domain. As the optimization advances, the algorithm gradually removes material from regions with low strain energy while reinforcing load-carrying paths, resulting in the emergence of a truss-like structure. By approximately iteration 40, the general topology is mostly defined, and subsequent iterations serve to refine the boundaries and eliminate intermediate densities. The final design at iteration 70 exhibits a clear black-and-white pattern with distinct structural members and minimal gray regions, indicating strong convergence and effective penalization through the SIMP method. This iterative development highlights the ability of the optimization framework to efficiently converge to a mechanically efficient design under volume constraints.

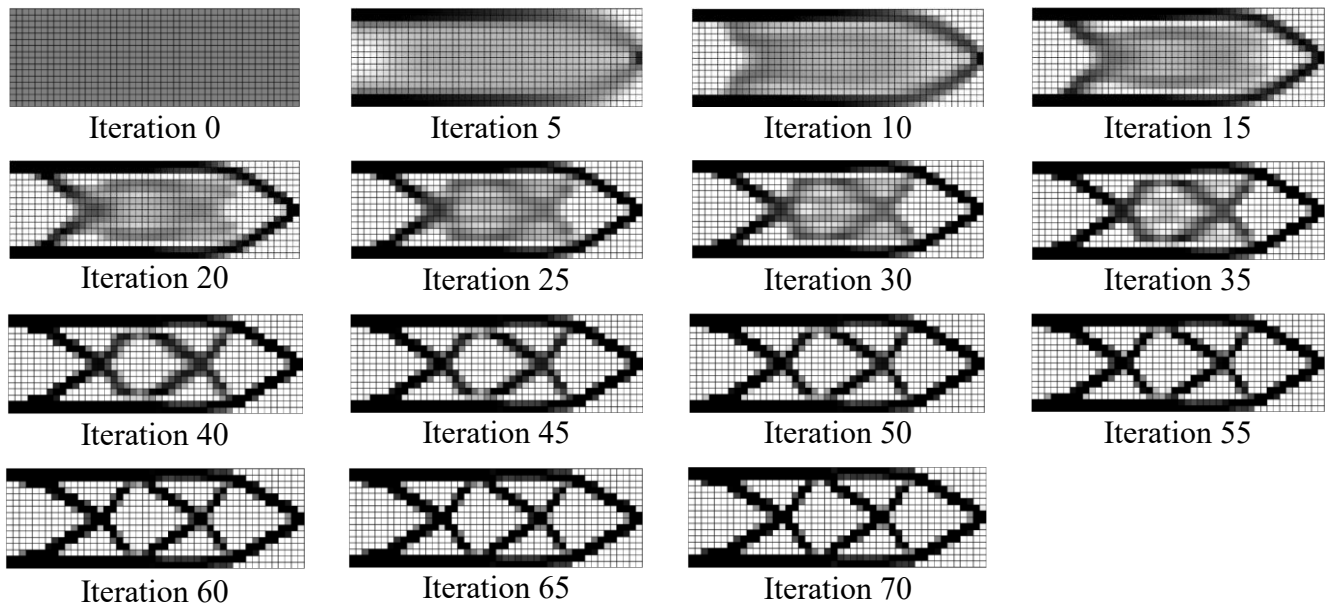


Figure 2. Evolution of the material distribution during the topology optimization process for the cantilever beam A (for $x_0 = 0.5$, $V_f = 0.4$).

Figure 3 presents the final optimized topology obtained after 70 iterations for the cantilever beam problem. The image shows the element-wise density distribution, where black represents solid material ($\rho \approx 1$) and white indicates voids ($\rho \approx 0$). Intermediate gray tones, which are minimal, represent partially penalized densities. The resulting structure forms a clear truss-like configuration with distinct diagonal and horizontal members, reflecting efficient load transfer paths from the point of application to the supports. The presence of only a few intermediate-density elements confirms the effectiveness of the penalization and filtering strategies in driving the design toward a nearly binary solution, which is desirable for manufacturability and structural clarity.

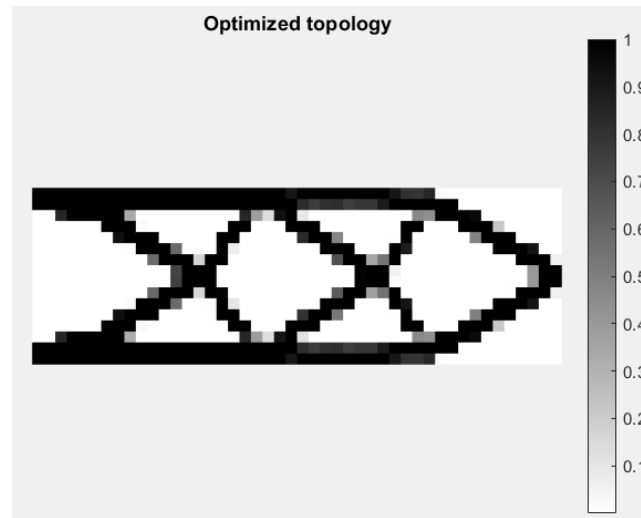


Figure 3. Final optimized topology of the cantilever beam A (for $x_0 = 0.5$, $V_f = 0.4$), visualized with element density distribution.

Investigation for Different Volume Fraction values

As the next step in our investigation, we extended the analysis by running the topology optimization algorithm for additional volume fraction values, specifically $V_f = 0.5$ and $V_f = 0.6$, in addition to the previously studied case of $V_f = 0.4$. The results of these cases are presented in Figure 4 and Figure 5, which summarize the performance of both optimized and unoptimized structures in terms of compliance and displacement.

Figure 4 shows the compliance versus volume fraction for the cantilever beam. The orange line represents the unoptimized designs, where the material is distributed uniformly across the domain. In contrast, the blue line corresponds to the results after topology optimization, where the material has been redistributed strategically to minimize compliance. As expected, the optimized structures consistently exhibit lower compliance values for the same amount of material, indicating enhanced stiffness and structural efficiency.

Similarly, Figure 5 illustrates the displacement at the load application point as a function of volume fraction. Again, the optimized designs (blue line) show significantly lower displacements compared to the unoptimized ones (orange line). For instance, at $V_f = 0.4$, the displacement of the unoptimized structure is 0.0297 mm, whereas the optimized structure achieves a displacement of only 0.0214 mm. This represents a 28% reduction in displacement while using the same amount of material, clearly demonstrating the effectiveness of the optimization process.

These results highlight the benefit of topology optimization in achieving mechanically superior designs under material constraints. As the volume fraction increases, both compliance and displacement decrease in both cases, but the optimized solutions consistently outperform

the uniform material distributions, confirming the value of intelligent material placement in structural design.

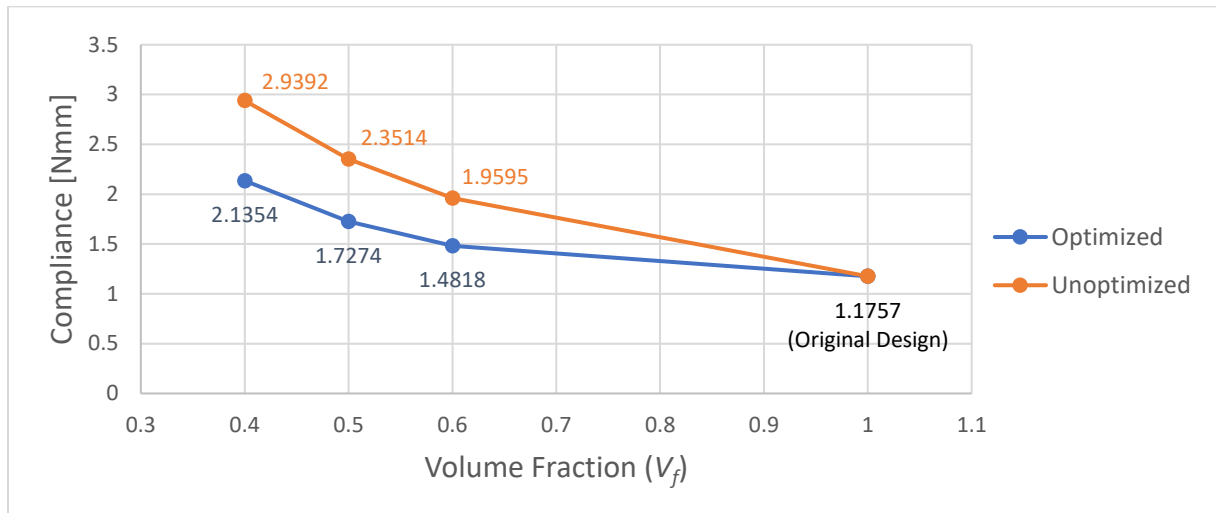


Figure 4. Compliance vs. volume fraction for the cantilever beam A example.

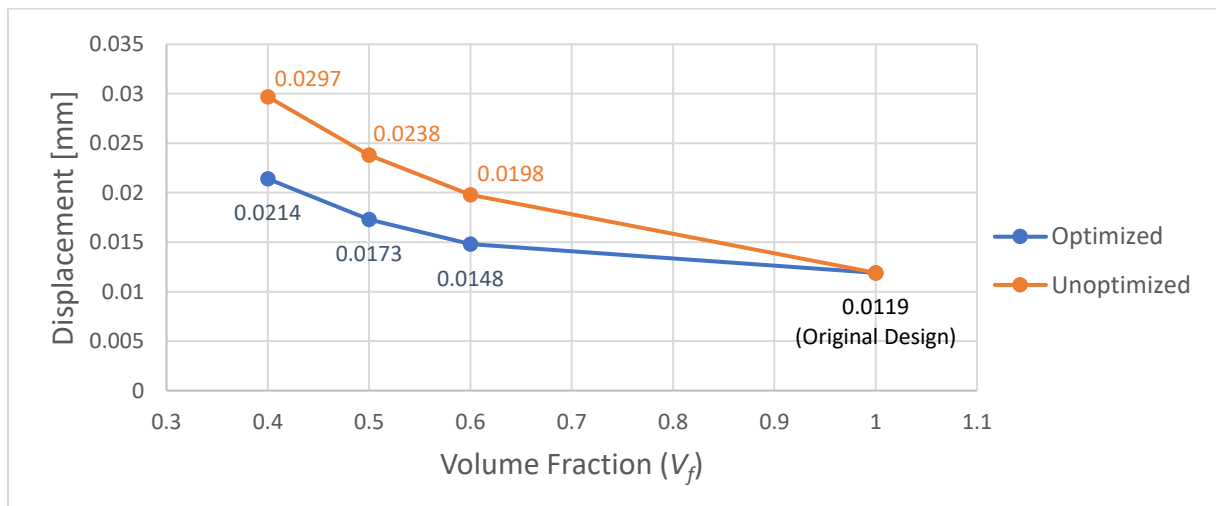


Figure 5. Displacement vs. volume fraction for the cantilever beam A example.

Verification Study with Autodesk Fusion

To validate the accuracy of the results obtained with our MATLAB-based topology optimization framework, a verification study was conducted using Autodesk Fusion's built-in topology optimization module. The same cantilever beam geometry, material properties, boundary conditions, and volume fraction constraint ($V_f = 0.4$) were used in the Fusion model. Since Fusion operates in a 3D environment, a small out-of-plane thickness of 1 mm was applied to approximate a 2D structural behavior, avoid the use of too many elements, and maintain consistency with the plane stress assumption in our framework. The resulting optimized topology from Fusion is shown in Figure 6, and it closely resembles the topology obtained with our MATLAB implementation (Figure 3). Both designs exhibit a clear truss-like layout with similar load paths, material distributions, and connectivity patterns. This strong agreement confirms the validity of our numerical implementation and demonstrates that the proposed framework

can produce reliable and physically meaningful results that are consistent with those generated by commercial software tools.

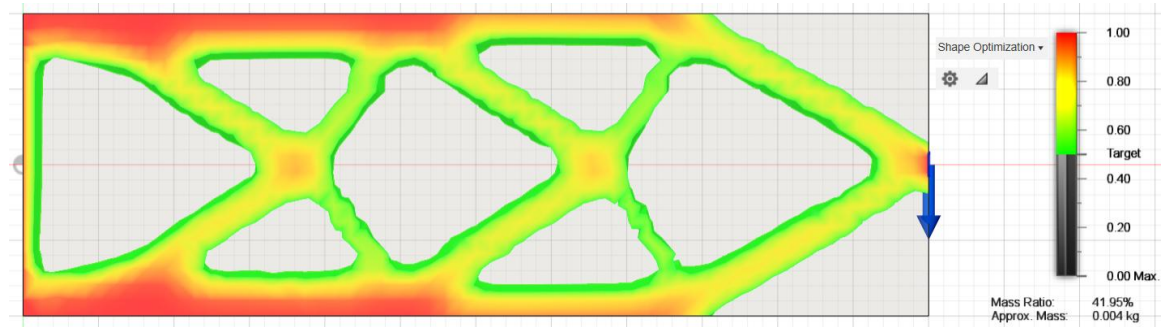


Figure 6. Final optimized topology obtained using Autodesk Fusion 360 for the cantilever beam A, under static loading.

4.2 Cantilever Beam B: Dynamic Analysis Case

Figure 7 shows the geometry of the cantilever beam B used in the dynamic topology optimization problem. The beam has a length $L = 8$ m, a height $H = 4$ m, and is fixed along its left vertical edge. As part of the dynamic analysis setup, a lumped mass m_c is added at the mid-height of the right edge, as indicated in the figure. In this formulation, there are no external loads applied to the structure. The elements themselves contribute to the mass of the system based on their material density and geometry.

It has to be noted that in the absence of a concentrated mass ($m_c = 0$), the optimizer tends to place material along the top and bottom fibers of the beam to maximize stiffness (i.e., to minimize the first eigenperiod). While this outcome is mathematically valid, it results in a design with two disconnected structural paths, which is undesirable from a practical engineering perspective. To address this issue, we follow the approach suggested in the literature [41, 42], and introduce the lumped mass m_c at a specific location to guide material placement. This additional mass acts as a surrogate for a potential load and encourages the optimizer to allocate material around the mass to support its influence on the vibration mode. This adjustment leads to more realistic and structurally continuous topologies, as will be demonstrated in the results section.

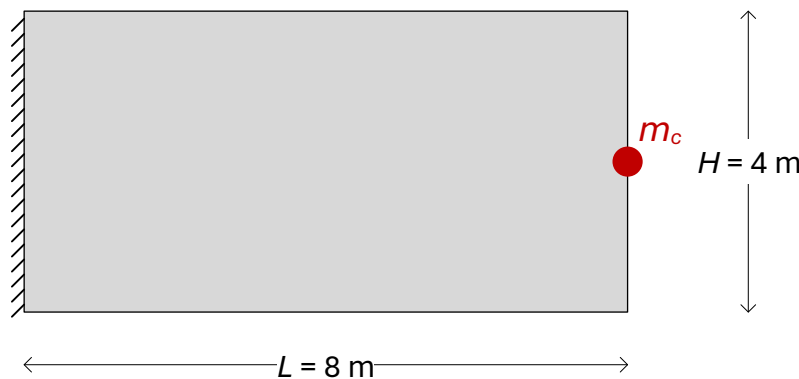


Figure 7. Cantilever beam B, used in the dynamic case example [41, 42].

In this example, all quantities are expressed using a consistent unit system: meters (m) for distance, newtons (N) for force, megagrams (Mg) for mass, and seconds (s) for time. These

units ensure dimensional consistency throughout the analysis. Material and other properties are given below [41]:

- Density: $\rho = 1000 \text{ kg/m}^3 = 1 \text{ Mg/m}^3$,
- Young's modulus: $E = 10 \text{ GPa} = 10 \times 10^6 \text{ kPa}$,
- Poisson's ratio: $\nu = 0.3$,
- Thickness: $t = 1 \text{ m}$.

The mesh adopted for this study comprises 24×12 elements, yielding a total of 288 elements and thus 288 design variables for the dynamic topology optimization problem. The finite element discretization results in 325 nodes and 650 total degrees of freedom (DOFs). With 13 nodes fully constrained along the left (fixed) edge, the number of active (free) DOFs is reduced to 624. The topology optimization parameters are as follows:

- Volume fraction: $V_f = 0.4$,
- SIMP penalty for stiffness matrix: $p = 3$,
- SIMP penalty for mass matrix: $q = 1$,
- Filter radius: $R = 0.66 \text{ (m)}$,
- Initial material distribution: Uniform, with $x_0 = 0.5$ (Initial volume fraction is 0.5 for the starting design).

The result of the dynamic topology optimization for the cantilever beam is shown in Figure 8, after 250 iterations. The structure has been optimized to maximize the fundamental eigen-frequency while satisfying a volume constraint of $V_f = 0.4$. The material and analysis settings are consistent with those used in related studies [41].

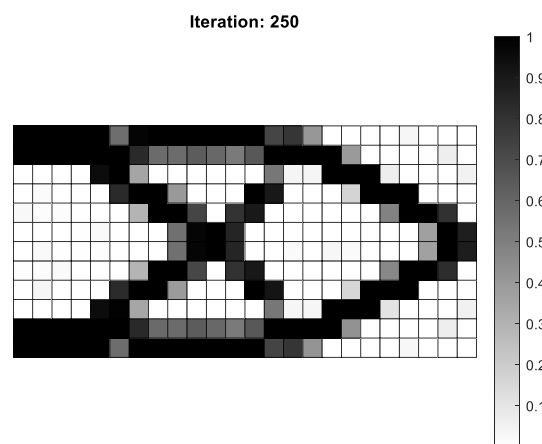


Figure 8. Final optimized topology for the dynamic problem of the cantilever beam B (for $x_0 = 0.5$, $V_f = 0.4$), visualized with element density distribution.

The resulting topology exhibits a clear truss-like structure that effectively supports the dynamic behavior influenced by the added mass. This design avoids the formation of separated top and bottom beams—a common issue when no lumped mass is introduced—and instead distributes material in a structurally meaningful way to carry the inertial effects. Our result is in very good agreement with the findings of Yaghoobi and Hassani [41], who studied the same cantilever configuration with an attached concentrated mass. Their optimal design closely

resembles ours in overall structural layout and material distribution. Both studies confirm that the inclusion of a concentrated mass significantly influences the optimization outcome, leading to practical and dynamically efficient topologies.

5 CONCLUSIONS

This study presented a unified MATLAB-based framework for topology optimization of two-dimensional structures, addressing both static and dynamic performance objectives using the SIMP method and a gradient-based optimization approach. The framework was implemented in a modular manner, allowing flexibility and extensibility for future enhancements. Static optimization was formulated as compliance minimization under a volume constraint, while dynamic optimization focused on maximizing the fundamental eigenfrequency (equivalently minimizing the first eigenperiod) without the need for externally applied loads.

For the static case, the framework successfully produced efficient, mechanically sound topologies that clearly outperformed unoptimized designs, as demonstrated by reductions in compliance and displacement for the same volume fractions. For the dynamic case, we implemented a formulation based on eigenvalue analysis, supported by the consistent mass matrix and a penalized density interpolation. The introduction of a concentrated lumped mass proved essential in steering the optimizer toward realistic and connected structural forms, avoiding topologically disconnected regions that are common in pure frequency-based objectives.

The numerical results confirmed the robustness and effectiveness of the proposed approach. In particular, the final optimized topology for the dynamic case showed strong agreement with previously published results from the literature, validating the accuracy and reliability of the developed code. These findings underscore the importance of combining penalization, density filtering, and targeted mass placement when optimizing dynamic structural behavior.

Overall, this work demonstrates that SIMP-based topology optimization can be efficiently implemented within MATLAB to solve both static and dynamic structural problems. The framework serves as a valuable educational and research tool and provides a solid foundation for future extensions to include different objectives, three-dimensional analysis, multi-objective formulations, stress constraints, nonlinear material behavior, and integration with AI-driven or generative design methods.

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