



Neurocomputing strategies for solving reliability-robust design optimization problems

Neurocomputing
strategies

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Abstract

Purpose – This paper, by taking randomness and uncertainty of structural systems into account aims to implement a combined reliability-based robust design optimization (RRDO) formulation. The random variables to be considered include the cross section dimensions, modulus of elasticity, yield stress, and applied loading. The RRDO problem is to be formulated as a multi-objective optimization problem where the construction cost and the standard deviation of the structural response are the objectives to be minimized.

Design/methodology/approach – The solution of the optimization problem is performed with the non-dominant cascade evolutionary algorithm with the weighted Tchebycheff metric, while the probabilistic analysis required is carried out with the Monte Carlo simulation method. Despite the computational advances, the solution of a RRDO problem for real-world structures is extremely computationally demanding and for this reason neurocomputing estimations are implemented.

Findings – The obtained estimates with the neural network predictions are shown to be very satisfactory in terms of accuracy for performing this type of computation. Furthermore, the present numerical results manage to achieve a reduction in computational time up to four orders of magnitude, for low probabilities of violation, compared to the conventional procedure making thus feasible the reliability-robust design optimization of realistic structures under probabilistic constraints.

Originality/value – The novel parts of the present work include the implementation of neurocomputing strategies in RRDO problems for reducing the computational cost and the comparison of the results given by RRDO and robust design optimization formulations, where the significance of taking into account probabilistic constraints is emphasized.

Keywords Neural nets, Optimization techniques, Structural design, Structural engineering

Paper type Research paper

1. Introduction

The primal engineering task during the design of any structural system is to minimize its construction cost and improve its structural performance. Improvements during the design stage can be achieved either by simply using design rules based on experience, or by an automated way using structural optimization procedures. Taking into account the complexity of a structural optimization problem it is obvious that finding the global optimum solution is not an easy task. In real-world conditions the significance of any “optimum” solution would be limited if the uncertainties involved in the geometric and material description of the structure as well as the loading conditions are not taken into account. This is because real-world structures have always imperfections which induce

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deviations from the nominal state assumed by the design codes. The deterministic optimum that has been obtained through a formulation that ignores scatter of any kind of the parameters affecting its response is never materialized in an absolute way; thus a near optimal solution is always applied in practice. Consequently, a deterministically optimum design may result in an infeasible design due to unavoidable scattering of the values of its basic parameters affecting its structural response. This happens because the performance of the applied “optimum” design may be far worse than expected. In order to account for the randomness of the parameters that affect the response of the structure a different formulation of the optimization problem has to be introduced, based on probabilistic treatment of the uncertainties involved.

The development of probabilistic analysis methods over the last two decades has stimulated the interest for considering randomness and uncertainty in the formulation of structural optimization problems (Schuëller, 2005). The probabilistic-based design optimization methodologies can be classified in the following formulations: robust design optimization (RDO) (Anthony and Keane, 2003; Beyer and Sendhoff, 2007; Doltsinis *et al.*, 2005; Lagaros and Papadrakakis, 2007; McAllister and Simpson, 2003; Park *et al.*, 2006; Parkinson *et al.*, 2007; Rangavajhala *et al.*, 2007; Ray and Smith, 2006; Schumacher and Olschinka, 2008; Zang *et al.*, 2005) and reliability-based design optimization (RBO) (Acar and Haftka, 2007; Agarwal and Renaud, 2004; Du *et al.*, 2006; Frangopol and Maute, 2003; Gunawan and Papalambros, 2006; Jensen *et al.*, 2008; Jiang *et al.*, 2000; Lagaros *et al.*, 2008; Liang *et al.*, 2007; Moses, 1997; Nikolaidis, 2007; Pu *et al.*, 1997; Rackwitz, 2004; Repalle and Grandhi, 2005; Royset *et al.*, 2001; Sørensen and Tarp-Johansen, 2005; Streicher and Rackwitz, 2004; Taflanidis and Beck, 2008). RDO methods primarily seek to minimize the influence of random variations of the nominal structural dimensions, material parameters and loading on the response of the structure. On the other hand, the main goal of RBO methods is to find the optimum design, which at the same time satisfies the objective of the minimum weight in conjunction with limitations on the allowable probability of failure or probability of exceedance of certain characteristic structural response quantities. The recent advances in RBO and RDO structural optimization problems can be found in the book by Tsompanakis *et al.* (2008).

In this work the reliability-based robust design optimization (RRDO) formulation, first addressed by Youn and Choi (2004) and Youn *et al.* (2007), is implemented in order to account for the influence of the probabilistic constraints in the framework of RDO of real-world structures. The objective functions considered in both RDO and RRDO formulations are the initial construction cost and the variance of the response of the structure, while in the RRDO formulation each design is checked whether it satisfies the provisions of the European design codes for steel structures (Eurocode 3) (CEN, 1993) with a prescribed probability of violation. The uncertainty of loads, material properties and cross section dimensions are taken into consideration and reliability analysis is performed with the Monte Carlo simulation (MCS) method combined with the latin hypercube sampling (LHS) technique. The optimization problem is dealt with the non-dominant cascade evolutionary algorithm (CEA) with the weighted Tchebycheff metric. However, despite the advanced features of the optimization formulation, the solution of a RDO problem considering probabilistic constraints, in addition to the deterministic ones, is an extremely computationally demanding task. In order to make the application of this optimization procedure feasible to real-world problems, artificial intelligence methodologies and in particular neural networks (NNs) are implemented to replace the conventional finite element analyses required by the MCS. This optimization scheme has been developed and improved in a number of

previous publications (Lagaros *et al.*, 2005a, 2008; Papadrakakis and Lagaros, 2002) into a robust and efficient, in terms of accuracy, methodology for treating optimization problems considering uncertainties.

2. The combined reliability-robust structural design optimization problem

A structural RDO problem is formulated as a two objective optimization problem where an additional, to the initial construction cost, objective function is considered related to the influence of the random nature of structural parameters and loading conditions on the performance of the structure (Lagaros and Papadrakakis, 2007). The aim is to minimize both the construction cost and the variance of the response of the structure. In the combined reliability-robust design optimization formulation, the probability of failure of the structure or the probability of violation of the constraints is taken into account as an additional set of probabilistic constraints together with the deterministic constraints imposed by the RDO formulation. Thus, the mathematical description of the generic RRDO problem implemented in this study is given as follows:

$$\begin{aligned} \min_{\mathbf{x} \in F} \quad & [C_{IN}(\mathbf{x}), \sigma_u(\mathbf{x}, \underline{\mathbf{r}})]^T \\ \text{subject to} \quad & g_j(\mathbf{x}) \leq 0, \quad j = 1, \dots, k \\ & p_{v, \max}(\mathbf{x}, \underline{\mathbf{r}}) \leq p_{v, \text{all}} \\ & x_i \in R_i^d, \quad i = 1, \dots, n \end{aligned} \quad (1)$$

where F is the feasible region, defined as the design variables space for which the deterministic constraints of Equation (1) are satisfied; $C_{IN}(\mathbf{x})$ and $\sigma_u(\mathbf{x}, \underline{\mathbf{r}})$ are the two objectives to be minimized, corresponding to the initial structural cost and the standard deviation of the structural response, respectively; $\mathbf{x}$ and $\underline{\mathbf{r}}$ are the vectors of the design and random variables; $g_j(\mathbf{x})$ are the deterministic constraint functions; $p_{v, \max}(\mathbf{x}, \underline{\mathbf{r}})$ is the maximum probability of violation, among the deterministic constraint functions; R_i^d is a discrete set from which the design variable x_i takes values and $p_{v, \text{all}}$ is the allowable probability of violation of the constraints. In the above formulation, an underlined symbol ($\underline{r}$) denotes a random variable, as opposed to its realization (r).

In this study three types of behavioural constraints are imposed for the steel structures considered:

- (1) stress;
- (2) compression force (for buckling); and
- (3) displacement constraints.

The stress constraint can be written as follows:

$$\begin{aligned} \sigma_{\max} &\leq \sigma_a \\ \sigma_a &= \frac{\sigma_y}{1.10} \end{aligned} \quad (2)$$

where $\sigma_{\max}$ is the maximum axial stress of each element group for all load cases, σ_a is the allowable axial stress according to Eurocode 3 (CEN, 1993) and σ_y is the yield stress. For members under compression an additional constraint is used for buckling:

$$\begin{aligned}
|P_{c,\max}| &\leq P_{cc} \\
P_{cc} &= \frac{P_e}{1.05} \\
P_e &= \frac{\pi^2 EI}{L_{\text{eff}}^2}
\end{aligned} \tag{3}$$

where $|P_{c,\max}|$ is the maximum axial compression force for all load cases, P_e is the critical Euler buckling force taken as the first buckling mode of a pin-connected member, L_{eff} is the effective length which for truss elements is taken equal to the actual length of each member, while E and I are the material modulus of elasticity and the cross section moment of inertia, respectively. Similarly, the displacement constraints can be written as:

$$|d| \leq d_a \tag{4}$$

where d is the displacement at a certain node or the maximum nodal displacement for all load cases and d_a is the allowable value of the displacement at a certain node or the maximum allowable nodal displacement.

3. The non-dominant CEA

A number of methods have been presented in the past dealing with multi-objective optimization problems with deterministic constraints (Coello Coello, 2000; Marler and Arora, 2004). These methods can be classified into:

- methods with a priori articulation of preferences;
- methods with a posteriori articulation of preferences; and
- methods with no articulation of preferences.

The multi-objective algorithm employed in this work is a hybrid method belonging to the first category, where an evolutionary algorithm is combined with a scalarizing function (Lagaros *et al.*, 2005b). According to the methods with a priori articulation of preferences, where a scalarizing function is employed, the multi-objective optimization problem is modified into a set of single objective optimization sub-problems. In general, when using scalarizing functions, local Pareto optimal solutions are obtained. Global Pareto optimality can be guaranteed only when the objective functions and the feasible region are both convex, or quasi-convex and convex, respectively. For non-convex cases, such as the majority of structural multi-objective optimization problems, a global single objective optimizer is required. CEA was found to be a suitable choice for dealing with such optimization problems (Charmpis *et al.*, 2005).

3.1 The cascade optimization concept

It is generally accepted that there is still no unique optimization algorithm capable of handling, with equal efficiency, all types of optimization problems. Cascade optimization attempts to alleviate this deficiency by applying a multi-stage procedure in which various optimizers are implemented successively. In the first stage of the cascade procedure the initial optimizer starts from a user specified design known as “cold-start”. The intermediate optimal solution reached in the first cascade stage, which may be perturbed using a pseudo-random technique, is called a “hot-start” and is used to initiate the second optimization stage. Accordingly, each optimization stage

of the cascade procedure starts from the (possibly perturbed) optimum design achieved in the previous stage. Thus, each cascade stage initiates from a hot-start and produces a new hot-start for the next stage. This way the autonomous computations of successive optimization stages are coupled. In general, the optimization algorithm implemented at each stage of a cascade process may or may not be the same. Cascade optimization has been implemented using different deterministic optimizers in the cascade stages, as well as using both deterministic and probabilistic optimizers one after another. The main argument in combining different optimizers in a successive manner has to do with the exploitation of the advantages offered by various optimization algorithms and reducing the influence of the corresponding disadvantages on the optimum design achieved. The selection of the optimizers to be included in a cascade procedure, their exact sequence and the number of cascade stages performed can be determined via a trial-and-error process.

In this work the idea of cascading is implemented within the evolution algorithm (EA) context for solving multi-objective structural optimization problems. In particular, the CEA method is employed for the solution of the sequence of parameterized single objective optimization problems. The proposed method is based on the idea of decomposing the multi-objective optimization problem into a series of $nruns$ single objective sub-problems which are solved concurrently with the CEA method since the sub-problems are independent with each other. CEA consists of $csteps$ optimization stages, each of which employs the same optimizer. In order to differentiate the search paths followed by the same optimization algorithm during the cascade stages, the initial conditions of individual optimization runs are suitably controlled by using at each stage:

- a different initial design (each stage initiates from the solution of the previous stage); and
- a different seed for the random number generator of the EA process.

The cascade procedure is terminated when no more improvement on the optimum solution vector can be attained over a small number of optimization stages.

3.2 Solving the multi-objective optimization problem

In this work the non-dominant CEA is further enhanced with the augmented Tchebycheff metric (Lagaros *et al.*, 2005b) for solving the multi-objective optimization problem as defined by the RRDO formulation of Equation (1). A detailed description of the algorithm can be found in the previous work of the authors (Lagaros *et al.*, 2005b). This methodology is abbreviated as $CEATm(\mu + \lambda)nruns, csteps$, where μ , λ are the number of the parent and offspring vectors used in CEA, $nruns$ is the number of independent CEA runs and $csteps$ is the number of cascade stages employed. A schematic description of the method is given in Figure 1. Following this enhancement, the objective function to be minimized for each sub-problem is the weighted Tchebycheff metric defined as follows:

$$\min_{\mathbf{x} \in \mathbf{F}} \max \left\{ \begin{aligned} &w_1 \frac{|C_{IN}(\mathbf{x}) - z_{CIN}^*|}{C_{IN}(\mathbf{x})} + \rho \left[\frac{|C_{IN}(\mathbf{x}) - z_{CIN}^*|}{C_{IN}(\mathbf{x})} + \frac{|\sigma_u(\mathbf{x}, \mathbf{r}) - z_{\sigma u}^*|}{\sigma_u(\mathbf{x}, \mathbf{r})} \right], \\ &w_2 \frac{|\sigma_u(\mathbf{x}, \mathbf{r}) - z_{\sigma u}^*|}{\sigma_u(\mathbf{x}, \mathbf{r})} + \rho \left[\frac{|C_{IN}(\mathbf{x}) - z_{CIN}^*|}{C_{IN}(\mathbf{x})} + \frac{|\sigma_u(\mathbf{x}, \mathbf{r}) - z_{\sigma u}^*|}{\sigma_u(\mathbf{x}, \mathbf{r})} \right] \end{aligned} \right\} \quad (5)$$

where $C_{IN}(\mathbf{x})$ and $\sigma_u(\mathbf{x}, \mathbf{r})$ are the two objective functions to be minimized; z_{CIN}^* , $z_{\sigma u}^*$ are the utopian values of the two objective functions corresponding to the optimum values

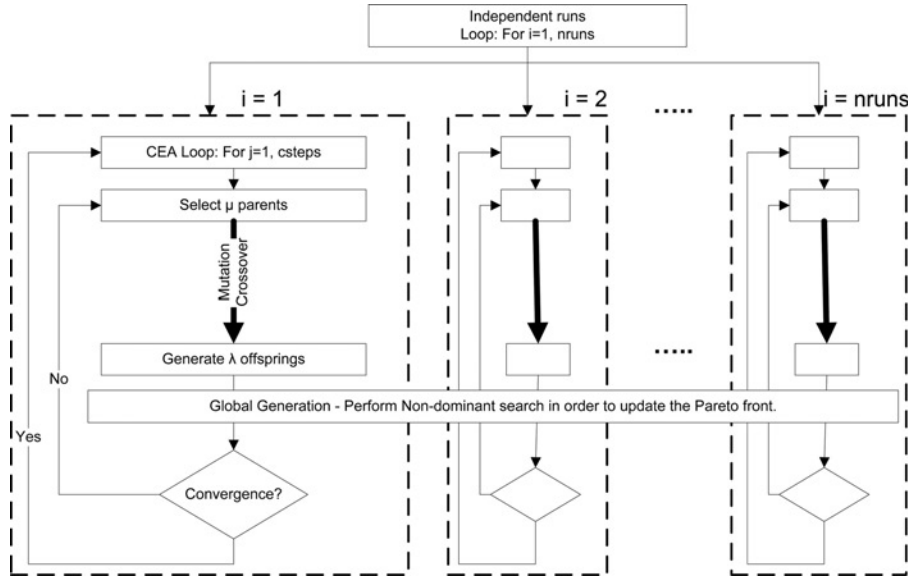


Figure 1.
The cascade evolutionary
algorithm (CEATm)

obtained when the problem is dealt with as a single objective optimization problem for each objective function; w_1, w_2 are the weight parameters assigned to the two objective functions and ρ is a sufficiently small positive scalar (in this work $\rho = 0.1$). The weight parameters are random numbers uniformly distributed in the range $[0, 1]$ fulfilling the requirement:

$$w_1 + w_2 = 1 \quad (6)$$

In order to ensure that the Pareto fronts will be composed by feasible designs, all the infeasible ones are rejected. The weight coefficients w_1, w_2 of Equation (5) are implemented in order to achieve convergence of the CEA runs to different directions of the design space. The Pareto front is computed by taking advantage of the search procedure performed for the independent CEA runs in these directions. The idea is to define the Pareto optimal solution reached so far in certain periods of the optimization procedure. These periods are called global generations. A global generation, shown in Figure 1, is completed when a population of $\mu + \lambda$ feasible designs is obtained for each sub-problem. Therefore, the temporary Pareto front is updated by performing non-dominant search over the global generation (i.e. over a suit of $\mu + \lambda$ by $nruns$ feasible designs) and the previous temporary Pareto set. A local Pareto front is computed in every global generation; the global Pareto front is reached when the optimization procedure converges.

4. Probabilistic analysis based on NN predictions

The formulation of the RRDO problem of Equation (1) requires the implementation of a reliability analysis in order to calculate $\sigma_u(\mathbf{x}, \mathbf{r})$ and $p_{v,max}(\mathbf{x}, \mathbf{r})$. Despite the improvements achieved on the efficiency of the computational methods for treating reliability analysis problems, they still require disproportional computational effort for practical reliability analysis problems. In probabilistic analysis of structures the MCS

method is particularly suitable when an analytical solution is not attainable. This is mainly the case of problems with complex nature and a large number of random variables where all other probabilistic analysis methods are not applicable.

4.1 The basic MCS method

One of the main tasks in structural probabilistic analysis problems is the calculation of the probability of violation of some behavioural constraints. Expressing the limit state function as $G(\mathbf{r}) < 0$, where $\mathbf{r} = [r_1, \dots, r_M]^T$ is a vector of the random structural parameters, the violation probability of the behavioural constraints can be written as:

$$p_{\text{viol}} = \int_{G(\mathbf{r}) \geq 0} f_r(\mathbf{r}) d\mathbf{r} \quad (7)$$

where $f_r(\mathbf{r})$ denotes the joint probability of violation for all random structural parameters.

Structural reliability analysis can be performed either with simulation methods, such as the MCS method, or with approximation methods. First- and second-order approximate reliability methods entail prior knowledge of the mean and the variance of each random variable, while a differentiable failure function is also required. On the other hand, MCS is simple to use and has the capability of handling practically every possible problem regardless of its complexity. Since MCS is based on the theory of large numbers (N_∞), an unbiased estimator of the probability of violation is given by:

$$p_{\text{viol}} = \frac{1}{N_\infty} \sum_{j=1}^{N_\infty} I(\mathbf{r}_j) \quad (8)$$

where $\mathbf{r}_j$ is the j -th vector of the random structural parameters, and $I(\mathbf{r}_j)$ is an indicator for successful and unsuccessful MCSs defined as:

$$I(\mathbf{r}_j) = \begin{cases} 1 & \text{if } G(\mathbf{r}_j) \geq 0 \\ 0 & \text{if } G(\mathbf{r}_j) < 0 \end{cases} \quad (9)$$

In order to estimate p_{viol} an adequate number of N independent random samples are produced. The value of the violation function is computed for each random sample $\mathbf{r}_j$ and the Monte Carlo estimation of p_{viol} is given in terms of sample mean by:

$$p_{\text{viol}} \cong \frac{N_H}{N} \quad (10)$$

where N_H is the number of simulations with constraint violation and N is the total number of simulations.

For typical structural reliability problems the computational effort involved in the basic MSC becomes excessive due to the enormous sample size required. Variance reduction techniques, such as importance sampling, directional simulation, antithetic variates or adaptive sampling, have been proposed in order to reduce the computational effort of MCS. The disadvantage of these methods is that they require prior knowledge of the structural behaviour in order to determine the most effective sampling region, which

for many practical problems is not clearly identifiable. Recent results (Koutsourelakis *et al.*, 2004) reveal that variance reduction techniques still require significant number of the system response evaluations to estimate failure probabilities of the order less than 10^{-3} . Other recently proposed simulation methods, such as line sampling (Koutsourelakis *et al.*, 2004) and subset simulation (Au and Beck, 2001), were proven to be very efficient in reducing the required sample size and the computational cost in certain types of problems; however, their performance and ergodicity are sensitive to the values of certain parameters which are not known *a priori*.

4.2 Latin hypercube sampling

The idea of the LHS technique was proposed by McKay *et al.* (1979) in an effort to reduce the required computational cost of random sampling methodologies. In the context of statistical sampling, a square grid containing sample positions is a Latin square if (and only if) there is only one sample in each row and each column. Figure 2 depicts an example of four samples in a two-dimensional space. The generalization of this concept to an arbitrary number of M dimensions constitutes a Latin hypercube of dimension M , where each sample is the only one in each axis-aligned hyperplane containing it.

Based on this concept, Latin hypercube samples are generated by dividing the range of M uncertain variables into N non-overlapping segments of equal probability. Thus, the M -dimensional parameter space is partitioned into N^M cells. For each random variable, a single value is randomly selected from each segment, producing a set of N values. In the case of a random variable r with probability density function $f(r)$, points $r_1, r_2, \dots, r_{N-1}$ divide its range $(-\infty, +\infty)$ into segments of equal probability $1/N$ as follows:

$$\int_{-\infty}^{r_1} f(r)dr = \int_{r_i}^{r_{i+1}} f(r)dr = \int_{r_{N-1}}^{+\infty} f(r)dr = \frac{1}{N}, \quad (i = 1, \dots, N-2) \quad (11)$$

The values of each random variable are randomly matched with each other to create N samples. The number of intervals N needs to be the same for all variables. The LHS method is independent of the random variables number as it does not require more samples for more random variables, which is one of its main advantages.

A schematic representation of LHS compared to the standard MCS is depicted in Figures 3(a), 3(b). In Figure 3(b), the total area of the distribution has been divided into $N=8$ regions, each one having an area of $1/8 = 0.125$. Eight samples have been generated for illustration, each one belonging to each region. It is clear that using the LHS, the distribution of samples is better and more representative of the distribution function, than using the standard MCS.

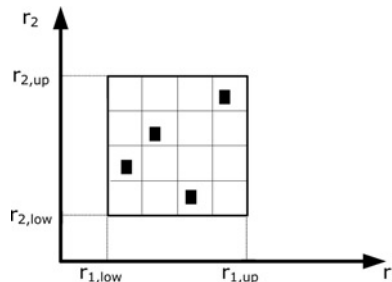
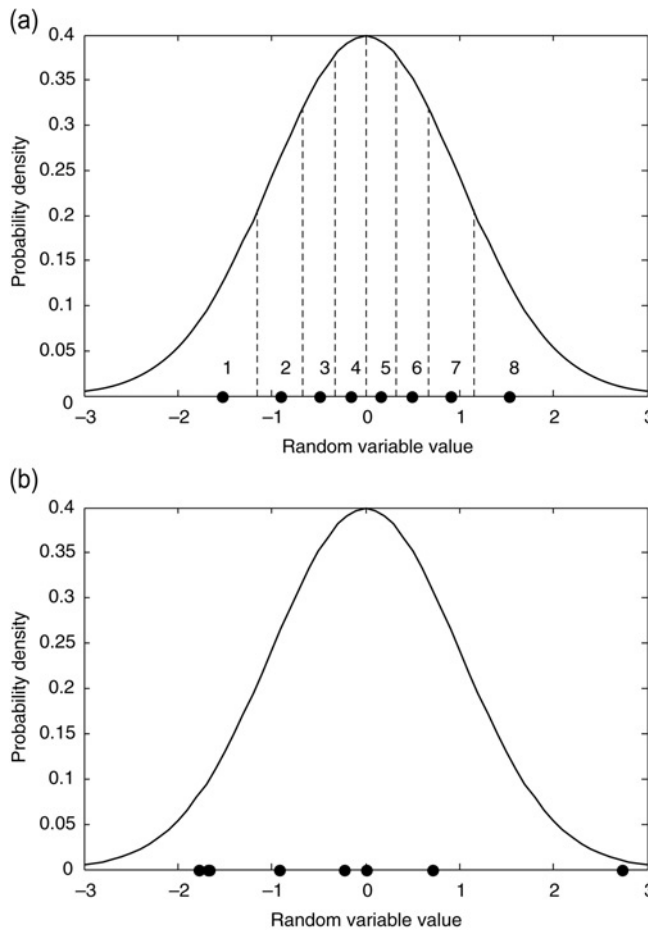


Figure 2.
Latin hypercube square:
two-dimensional space



Notes: (a) Standard MC sampling; (b) LHS ($N = 8$ segments of equal probability)

Figure 3.
Sampling techniques for
the normal distribution

The advantage of the LHS approach is that the random samples are generated from all ranges of possible values. LHS is generally recognized as one of the most efficient size reduction techniques for the calculation of statistical quantities and relatively large violation probabilities (Owen, 1997). LHS can improve the efficiency of MCSs by picking the input samples in a more effective way. Whereas MC methods typically pick points at random within the domain, latin hypercube samples the entire domain more systematically. It is a strategy for generating random sample points ensuring that all portions of the random space are properly represented. Thus, by means of LHS method, the whole parameter space can be sampled more reliably with fewer samples, improving convergence rates and speeding up the execution time.

4.3 NN-based MCS probabilistic analysis

Despite the improvements achieved by the LHS and other variance reduction techniques, MCS still requires disproportional computational effort for reliability analysis of realistic

problems. This is the reason why very few successful numerical investigations are known in estimating the probability of failure and are mainly restricted to simple elastic frames and trusses (Jiang *et al.*, 2007). In a previous work by the authors (Papadrakakis and Lagaros, 2002) a methodology has been proposed where NNs have been incorporated into MCS in order to reduce the computational cost required for solving real-world RBO problems. In this work this methodology is incorporated in the framework of RRDO problems. The accuracy of the predictions of an NN depends on several characteristics of the problem, like the number of input variables, the nonlinearity of the problem, among others. If the dimensionality of the problem is increased, which is the case when the problem is enhanced from a probabilistic point of view, the input nodes will increase and thus the accuracy will decrease. For the test cases considered in this study, as well as for other applications, the dimensionality of the probabilistic problem is adequate. The authors have successfully implemented neurocomputing strategies in a variety of structural problems ranging from linear static response of truss structures to the prediction of the nonlinear response under dynamic loading of steel and reinforced concrete buildings (Lagaros *et al.*, 2005a, 2008; Lagaros and Papadrakakis, 2009; Papadrakakis and Lagaros, 2003; Papadrakakis *et al.*, 1999, 1996).

The methodology, shown schematically with the flowchart of Figure 4, takes advantage of the global approximation capabilities of the NNs. For every new

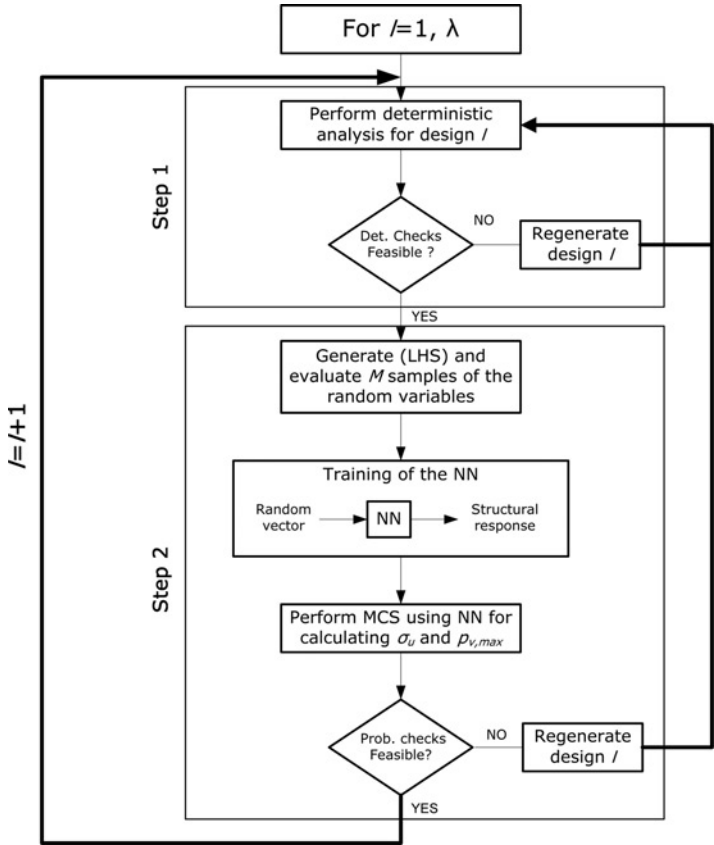


Figure 4.
The NN-assisted
MCS procedure

candidate Pareto optimum design, a new NN is trained to replace the finite element analyses required during the MCSs. Thus, the computational cost required for performing the probabilistic constraints' checks and calculating the statistical quantities needed by the RRDO formulation is reduced significantly as will be shown in the numerical results section.

Figure 4 describes the procedure for assessing the λ offsprings of an optimization cycle (generation) of CEA. After training, over a sample of N random vectors generated using LHS, the trained NN is applied as a predictor of the structural performance for every new vector of random variables encountered during the MCS. As soon as the NN is trained, the NN approximations of the response are defined according to the following equation:

$$\bar{u}(\mathbf{x}) = \mathbf{f}(\mathbf{w}^O \cdot \mathbf{f}(\mathbf{w}^H \cdot \mathbf{x} + \mathbf{b}^H) + \mathbf{b}^O) \quad (12)$$

where $\mathbf{w}^O$ and $\mathbf{w}^H$ are the matrices containing the weight parameters of the output and hidden layers, respectively; $\mathbf{b}^O$ and $\mathbf{b}^H$ are the vectors containing the bias parameters of the output and hidden layers, respectively, and $\mathbf{f}(y)$ is the sigmoid transfer function used for both layers of the NN:

$$\mathbf{f}(y) = \frac{1}{1 + \exp(-\gamma y)} \quad (13)$$

The coefficient γ defines the slope of the sigmoid function.

Extrapolation in NNs remains a problem despite the fact that special types of NNs have been proposed for this reason (Dariani *et al.*, 2007; Sari *et al.*, 2007). This is due to the property of NNs to map virtually any function by adjusting its parameters according to the presented training data. Hence it is important to have sufficient and properly distributed, over the range of interest, training data to avoid NN performing extrapolation instead of interpolation. This drawback is alleviated by generating the training sample using the LHS method in the range of $\mu_i \pm 6\sigma_i$ for any random variable i , where μ_i and σ_i are the mean value and standard deviation of the i -th random variable. This ensures that 99.999998 percent of the samples will be in this range (Koch *et al.*, 2004) and thus NN will always predict interpolated values. If during the NN-based MCS a sample is generated outside the prescribed range then this sample is rejected and a new one is generated.

5. Numerical results

The proposed methodology is tested and validated in two three-dimensional (3D) steel truss structures. For both test examples, two objective functions are considered: The initial construction cost and the standard deviation of a characteristic nodal displacement, representative of the structural response under the external loading. Two sets of constraints are enforced: deterministic constraints on stresses, element buckling and displacements imposed by the Eurocode 3 (CEN, 1993), and a probabilistic constraint on the maximum probability of violation of the deterministic constraints. Furthermore, due to manufacturing limitations the design variables are not considered continuous but discrete. The discrete design variables are treated in the same way as in single objective design optimization problems using the discrete version of evolution strategies (Papadrakakis *et al.*, 1998). The design variables considered are the dimensions of the members of the structure taken from the circular hollow section (CHS) table of the

Eurocode (CEN, 1993). The random variables related to the cross-sectional dimensions, for both test examples, are two per design variable: the external diameter D and the thickness t of the CHS. Apart from the cross-sectional dimensions of the structural members, the material properties (modulus of elasticity E and yield stress σ_y) as well as the lateral loads have been considered as random variables.

5.1 3D 39-bar truss

The first test example considered is the 39-bar truss structure shown in Figure 5. The FE model consists of 15 nodes and 39 elements divided into four groups. A vertical load $V=2\text{ kN}$ is applied to all nodes, while a probabilistic horizontal load $F=8\text{ kN}$ is applied to the top nodes along the x -direction. The type of probability density function, the mean value and the variance of the random variables are shown in Table I.

The NN configuration implemented has one hidden layer with 30 nodes resulting in a fully connected 11-30-4 NN architecture which is used for all runs. The 11 input nodes of the NN correspond to the random variables of Table I, which are the dimensions D and t of the four design variables, the modulus of elasticity and the allowable stress of the structural members and the value of the horizontal load. On the other hand, the four output nodes of the network correspond to the characteristic displacement, the maximum displacement, the maximum compressive force and the maximum tensile force.

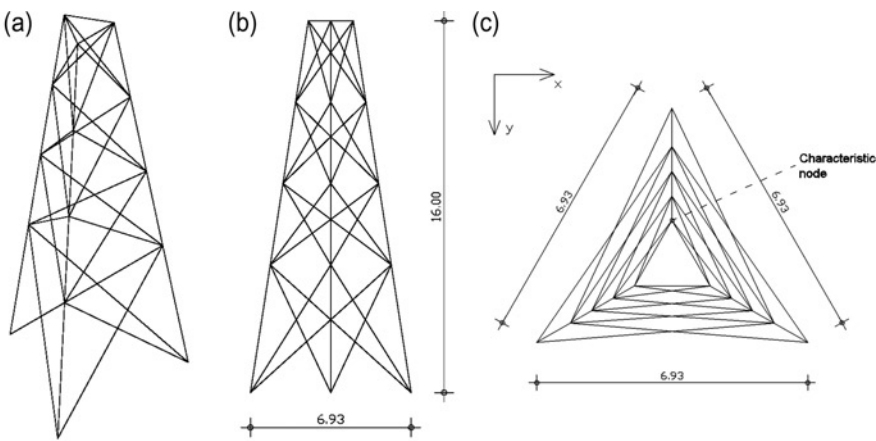


Figure 5.
39-bar truss

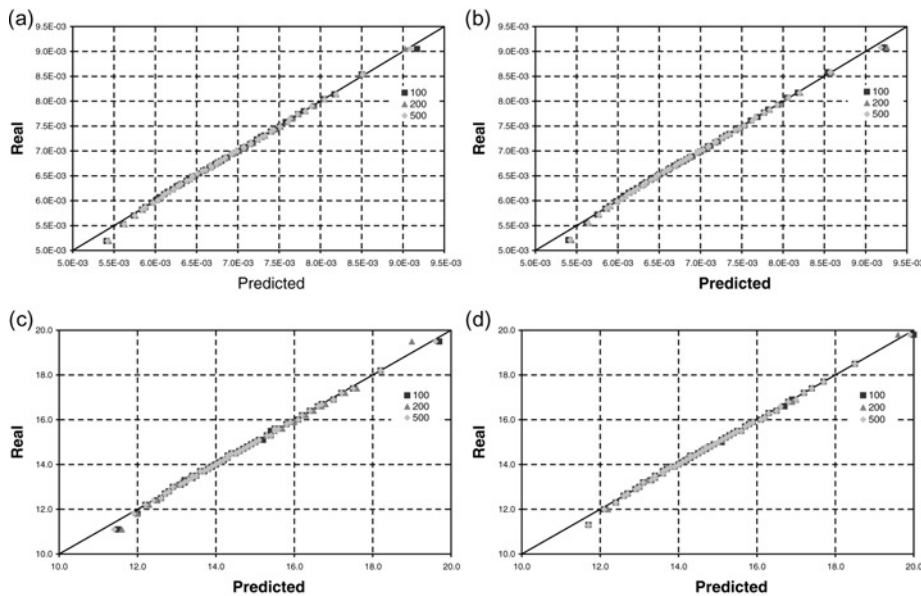
Notes: (a) 3D view; (b) side view; (c) top view (dimensions in meters)

Table I.
39-bar truss:
characteristics of the
random variables

		Probability density function	Mean value	Coefficient of variation (%)
E (kN/m ²)	Young's modulus	Normal	2.10×10^8	7.14
σ_y (kN/m ²)	Allowable stress	Normal	355,000	10.00
F (kN)	Horizontal loading	Normal	8	37.50
D	CHS diameter	Normal	D_i^a	2.0
t	CHS thickness	Normal	t_i^a	2.0

Note: ^aTaken from the CHS table of the Eurocode, for every design

The performance of the NN predictions is demonstrated in Figure 6(a)-(d), where the NN predicted values are plotted versus the real values for various number Σ of NN training patterns. Three different training sets, of size 100, 200 and 500, have been examined, all generated randomly using the LHS method, while 50 patterns (also randomly generated with LHS) have been used for testing the prediction capabilities of the trained NN over the entire range of interest. Due to the special care described in the previous section, to alleviate the possibility of NN extrapolation, as it can be seen in Figure 6(a)-(d), 100 patterns are adequate for training efficiently the NN. In order to check the accuracy of the NN predictions, the probability of violation was estimated using the same NN architecture trained with 100, 200 and 500 patterns and compared to the “exact” values as obtained by the conventional LHS-MCS. The results are shown in Table II for various numbers of MCSs where it can be seen that the “real” probability of violation is 0.53 percent, obtained for 5,000 or 10,000 MCSs. It is confirmed that 100



Notes: (a) Characteristic displacement; (b) maximum displacement; (c) maximum compressive force; (d) maximum tensile force (for the design: $\text{Sec}_1 = 139.7 \times 12.5$, $\text{Sec}_2 = 193.7 \times 16$, $\text{Sec}_3 = 193.7 \times 10$, $\text{Sec}_4 = 168.3 \times 10$)

Figure 6. 39-bar truss: performance of NN with respect to the number of the training patterns

No. of MCSs	“Exact”	NN predictions		
		100	200	500
		Probability of violation (%)		
500	1.00	1.40	1.20	1.40
1,000	7.00×10^{-1}	9.00×10^{-1}	8.00×10^{-1}	9.00×10^{-1}
2,000	5.50×10^{-1}	6.50×10^{-1}	6.00×10^{-1}	6.50×10^{-1}
3,000	5.00×10^{-1}	5.67×10^{-1}	5.33×10^{-1}	6.33×10^{-1}
5,000	5.30×10^{-1}	5.40×10^{-1}	5.20×10^{-1}	5.20×10^{-1}
10,000	5.30×10^{-1}	5.40×10^{-1}	5.20×10^{-1}	5.20×10^{-1}

Table II. 39-bar truss: accuracy study of the NN predictions for a training set of 100, 200 and 500 patterns

training patterns are adequate for an accurate prediction since a 0.54 percent probability of violation is obtained by the NN for 5,000 or 10,000 MCSs.

For the solution of the multi-objective optimization problem in question, the non-dominant $CEATm(\mu + \lambda)_{nrun, csteps}$ optimization scheme was implemented with $\mu = \lambda = 5$, $nrun = 10$ and $csteps = 3$. These values were found to be suitable for providing good quality Pareto front curves in a previous study of the authors (Lagaros *et al.*, 2005b). Four different formulations of the RDO problem have been considered:

- (1) the standard RDO formulation;
- (2) the RRDO_2 percent;
- (3) RRDO_0.1 percent; and
- (4) RRDO_0.01 percent.

with allowable probability of violation of the deterministic constraints equal to 2, 0.1 and 0.01 percent, respectively. The resultant Pareto front curves for the above-mentioned formulations are depicted in Figure 7, with the initial construction cost on the horizontal axis and the standard deviation of the characteristic node displacement on the vertical axis.

The Pareto front curve shown in Figure 7 represents solutions favouring one or the other objective function. Solutions near optimal design points *A* favour the objective function that is related to the initial cost, while solutions near the design points *B* favour the objective function that is related to the standard deviation of the response. All other intermediate points are compromise solutions between these two optimal solutions that are conflicting to each other. It can be seen in Figure 7 that the influence of the probabilistic constraint is significant near the optimum designs A_i ($i = 1, \dots, 4$). These designs are located in the Pareto front area dominated by the initial construction cost (C_{IN}) of the structure, where the four designs are substantially affected by the conditions imposed by the probabilistic constraint. It can be seen however, that the four Pareto front curves gradually converge as the importance of the second criterion (standard deviation of the response), increases.

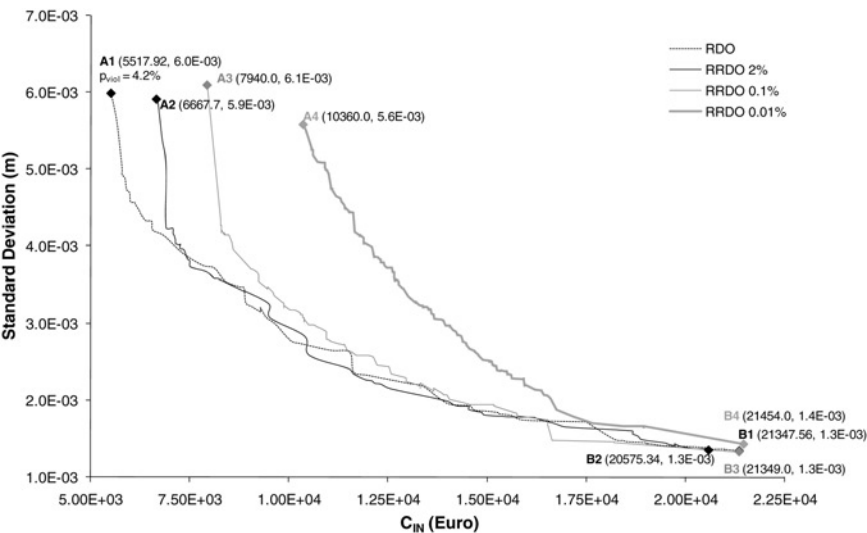


Figure 7.
39-bar truss: comparison
of the Pareto front curves

The optimization formulations considered differ in the sample size required for the MCS. For RDO and RRDO_2 percent formulations, a sample size of 10,000 was adequate, while for RRDO_0.1 percent and RRDO_0.01 percent formulations, a sample size of 100,000 and 500,000 was required, respectively. Different formulations and consequently different sample sizes lead to significantly different computing costs. The computational effort required for solving the optimization problem is given in Table III, where the conventional and the NN-based computing times are reported. It can be seen that the computational time is reduced by three orders of magnitude for RRDO_0.1 percent and RRDO_0.01 percent cases.

5.2 3D truss tower

The second test example considered is the 3D truss tower shown in Figures 8(a)-8(c) and is selected to demonstrate the performance, in terms of accuracy and computational efficiency, of the proposed optimization scheme for a rather large-scale problem. The FE model consists of 324 nodes and 1,254 elements which are divided into 12 groups of design variables. The applied loading consists of:

- self-weight (dead load);
- live loads; and
- wind actions according to the Eurocode 1 (CEN, 1991).

The type of probability density function, the mean value and the variance of the random variables are shown in Table IV. The horizontal displacement (in the x -direction) of the top node is selected as the characteristic displacement to be monitored (Figure 8(c)).

The NN configuration implemented has one hidden layer with 50 nodes resulting in a fully connected 27-50-4 NN architecture. The 27 input nodes of the NN correspond to the random variables of Table IV, which are the dimensions D and t of the twelve design variables, the modulus of elasticity and the allowable stress of the structural members and the value of the horizontal load. The performance of the NN is depicted in Figure 9, where the prediction of the characteristic displacement, the maximum displacement, the maximum compressive force and the maximum tensile force are shown. As it can be seen, 100 patterns are adequate for training efficiently the NN. In accordance to the previous test example, in order to test the accuracy of the NN predictions, the probability of violation is also assessed for the three patterns sizes. The obtained results are shown in Table V where it can be seen that 500 patterns give a more accurate prediction of the probability of violation compared to 100 or 200 patterns. The computing cost is given in Table VI, where the computing times of the conventional and the NN-based optimization procedures are reported. It has to be noted that the computing costs of the conventional RRDO_0.1 percent and RRDO_0.01 percent formulations are estimations due to the excessive computing cost required for

Formulation	No. of simulations	Time (h)	
		LHS-MCS	MCS-NN
RDO	10,000	1.81	2.17×10^{-2}
RRDO_2 percent	10,000	1.82	2.16×10^{-2}
RRDO_0.1 percent	100,000	1.76×10^1	2.12×10^{-2}
RRDO_0.01 percent	500,000	8.60×10^1	2.06×10^{-2}

Table III.
39-bar truss:
computational
efficiency study

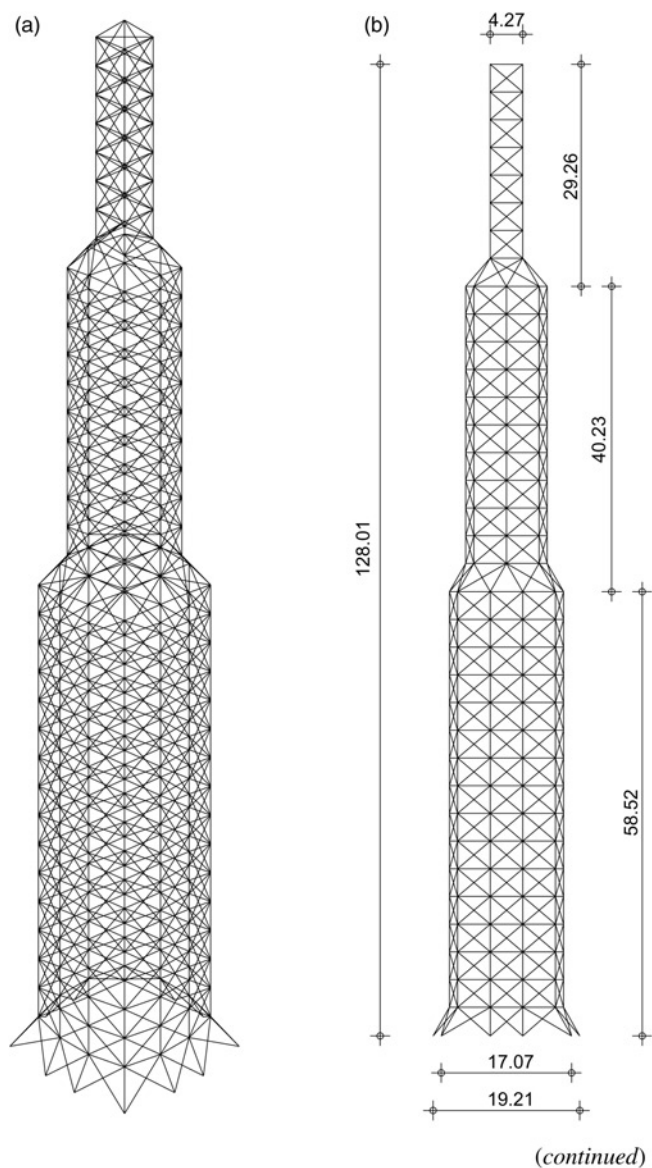
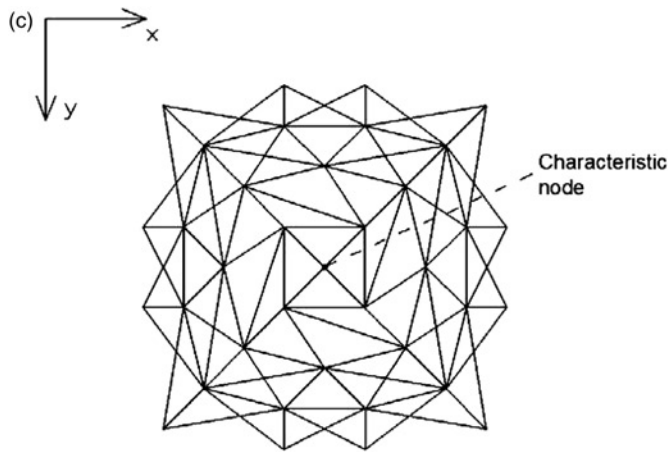


Figure 8.
3D truss tower

these two cases. It can be seen that the NN-based methodology requires up to three orders of magnitude less computing time compared to the conventional formulations. The optimization scheme used in the previous test example is also implemented here and the resultant Pareto front curves for the RDO formulations are depicted in Figure 10 with and without probabilistic constraints. Similar behaviour to the previous test example can be observed regarding the quality of the Pareto front curves and the dominant regions of each objective function.



Notes: (a) 3D view; (b) front and left views (dimensions in meters);
(c) top view

Figure 8.

		Probability density function	Mean value	Coefficient of variation (%)
E (kN/m ²)	Young's modulus	Normal	2.10×10^8	7.14
σ_y (kN/m ²)	Allowable stress	Normal	355,000	1,000
F (kN/m ²)	Wind loading	Normal	F_{μ}	4,000
D	CHS diameter	Normal	D_i^a	2.0
t	CHS thickness	Normal	t_i^a	2.0

Note: ^aTaken from the CHS table of the Eurocode, for every design

Table IV.
3D truss tower:
characteristics of the
random variables

6. Concluding remarks

The deterministic optimum is not always a “safe” design, since there are many random factors that affect the design during its lifetime. In order to find the “real” optimum the designer has to take into account all important random parameters. For this purpose two separate formulations have been proposed: RBO and RDO. However, due to the excessive computational demands required for addressing these type of problems, all studies were restricted to very simple academic examples. In the present work a reliability analysis combined with a RDO formulation is proposed where probabilistic constraints are incorporated into the formulation of the RDO problem.

The aim of this work in addressing a structural optimization problem considering uncertainties was twofold. First the influence of the probabilistic constraints was examined, and in particular those related to the probability of violation of the behavioural constraints, in addition to the variance of the structural response due to a number of uncertainties. In the second part an NN-based methodology was implemented for accelerating the computational efficiency of the proposed methodology and making the application of this type of optimization problems feasible for realistic structures.

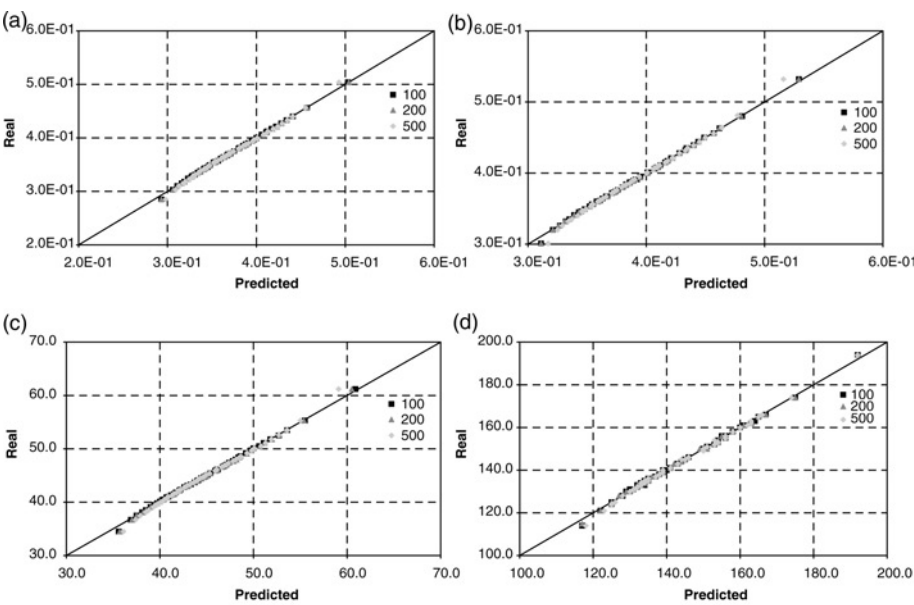


Figure 9.
3D truss tower:
performance of NN with
respect to the number
of the training patterns

Notes: (a) Characteristic displacement; (b) maximum displacement; (c) maximum compressive force; (d) maximum tensile force (for the design: $\text{Sec}_1 = 323.9 \times 16$, $\text{Sec}_2 = 193.7 \times 10$, $\text{Sec}_3 = 273 \times 20$, $\text{Sec}_4 = 219.1 \times 16$, $\text{Sec}_5 = 323.9 \times 7.1$, $\text{Sec}_6 = 323.9 \times 16$, $\text{Sec}_7 = 244.5 \times 20$, $\text{Sec}_8 = 273 \times 16$, $\text{Sec}_9 = 273 \times 10$, $\text{Sec}_{10} = 244.5 \times 16$, $\text{Sec}_{11} = 273 \times 7.1$, $\text{Sec}_{12} = 323.9 \times 8.0$)

Table V.
3D truss tower:
accuracy study of the
NN predictions for a
training set of 100, 200
and 500 patterns

No of MCSs	"Exact"	Probability of violation (%)		
		NN predictions		
		100	200	500
500	0.00	0.00	0.00	0.00
1,000	1.00×10^{-1}	2.00×10^{-1}	2.00×10^{-1}	1.00×10^{-1}
2,000	2.00×10^{-1}	2.50×10^{-1}	2.50×10^{-1}	2.00×10^{-1}
3,000	2.00×10^{-1}	2.33×10^{-1}	2.33×10^{-1}	2.00×10^{-1}
5,000	2.22×10^{-1}	2.67×10^{-1}	2.89×10^{-1}	2.44×10^{-1}
10,000	2.22×10^{-1}	2.67×10^{-1}	2.89×10^{-1}	2.44×10^{-1}

The combined reliability-robust design optimization formulation requires structural reliability analyses to be performed for every candidate design of the optimizer based on evolution strategies, in order to determine the maximum probability of violation of the constraints. Depending on the value of the allowable probability of violation, different sample sizes are used in order to calculate with an acceptable accuracy the statistical quantities under consideration, i.e. the standard deviation of the structural response and the probability of violation of the constraints. The Pareto front curves obtained for the combined reliability-robust design optimization and the RDO formulations are much different when the initial construction cost is predominant in

the objective function. Consequently, the optimum initial construction costs achieved by the combined reliability-robust design optimization formulations are larger than the corresponding initial construction cost achieved by the RDO.

It has been demonstrated that the solution of a combined reliability-robust design optimization of real-world problems in structural mechanics is an extremely computationally intensive task. Although the LHS has been implemented for improving the computational efficiency of the Monte Carlo method, the computational cost remains excessive making the solution of such problems computationally unsolvable. In this work an NN assisted methodology has been proposed in order to obtain estimates of the structural response required during the reliability analysis with the MCS method. The obtained estimates with the NN predictions have shown to be very satisfactory in terms of accuracy for performing this type of computations. Furthermore, the present numerical results manage to achieve a reduction in computational time up to four orders of magnitude, for low probabilities of violation, compared to the conventional procedure making thus feasible the reliability-robust design optimization of realistic structures under probabilistic constraints.

Formulation	No. of simulations	Time (h)	
		LHS-MCS	MCS-NN
RDO	10,000	5.33×10^1	3.08
RRDO_2 percent	10,000	5.42×10^1	3.12
RRDO_0.1 percent	100,000	5.59×10^{2a}	3.19
RRDO_0.01 percent	500,000	2.79×10^{3a}	3.50

Note: ^aEstimated

Table VI.
3D truss tower:
computational
efficiency study

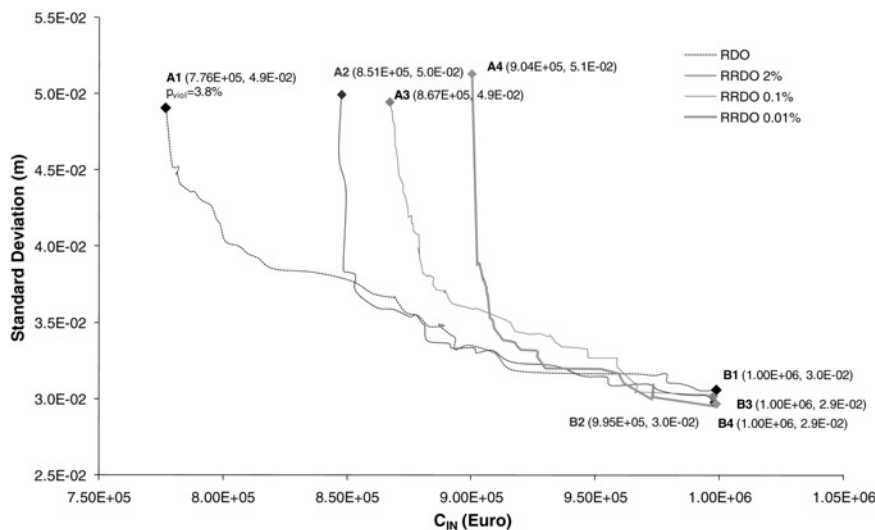


Figure 10.
3D truss tower:
comparison of the Pareto
front curves

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