

Modeling of masonry failure surface under biaxial compressive stress using Neural Networks



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HIGHLIGHTS

- Neural Networks are used in order to approximate the experimental results for masonry failure.
- A two-step procedure is proposed, with the training of two types of Neural Networks.
- The NNs showed great performance in fitting the experimental input data.
- The curves generated by the NNs are continuous and smooth, but not necessarily convex.

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ABSTRACT

Masonry is a brittle anisotropic material that exhibits distinct directional properties because the mortar joints act as planes of weakness. To define failure under biaxial stress, a 3D surface in terms of the two principal stresses and their orientation to the bed joints, is required. In the present study, a novel method is proposed on applying Neural Networks (NNs) to approximate the failure surface for such brittle anisotropic materials. The method comprises a series of NNs that are trained with available experimental data. The results demonstrate the great potential of using NNs for the approximation of masonry failure surface under biaxial compressive stress.

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1. Literature review

Masonry exhibits distinct directional properties due to the influence of the mortar joints. Depending upon the orientation of the joints to the stress directions, failure can occur in the joints alone or simultaneously in the joints and the blocks. The failure of masonry under uniaxial and biaxial stress state has been studied experimentally in the past by many researchers but to the authors' knowledge there have not been many attempts to apply a Neural Network (NN) for the prediction of masonry behavior in general.

NNs have been used in many ways for various other problems in the literature. Duan et al. [1] studied the applicability of NNs in modeling the elastic modulus of recycled aggregate concrete. In this study, an NN-I was first constructed by using 324 data sets collected from 21 international published works, which were

randomly divided into three groups as the training, testing and validation sets, respectively. Then a NN-II was also trained with 16 more data sets of the authors' own experimental results to examine whether the performance of the Artificial Neural Network (ANN) could be further improved.

In another work, Yurdakul and Akdas [2] tried to model the uniaxial compressive strength of building stones using non-destructive test results as Neural Networks input parameters. The uniaxial compressive strength value was used as a critical input parameter in determining the engineering properties of natural building stones. The purpose of the study was to develop a model to determine the strength of natural building stones via relatively simple and low-cost mechanical tests with the application of NNs.

Alexandridis et al. [3] proposed a Neural Network approach for compressive strength prediction in cement-based materials through the study of pressure-stimulated electrical signals. They presented a non-destructive method for predicting the compressive strength of cement-based materials by studying the appearance of weak electrical signals at specimens that are under

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mechanical stress. A series of lab experiments were conducted in order to record the pressure-stimulated electrical signals in cement mortar specimens.

In [4], Tanarslan et al. proposed the use of a NN model to predict the shear capacity of reinforced concrete beams, retrofitted in shear by means of externally bonded wrapped and U-jacketed fiber-reinforced polymer. Unlike the existing design codes the model considered the effect of strengthening configurations dissimilarity. In addition it also considered the effect of shear span-to-depth ratio at the ultimate state. Mechanical properties of strengthening material and mechanical and dimensional properties of beams were selected as inputs for the NN.

Mashrei et al. [5] proposed a Back-Propagation Neural Network (BPNN) model for predicting the bond strength of FRP-to-concrete joints. Published single-lap shear test specimens were used to predict the bond strength of externally bonded FRP systems adhered to concrete prisms. A database of one hundred and fifty experimental data points from several sources was used for training and testing the BPNN. The data used in the BPNN were arranged in a format of six input parameters including: width of concrete prism; concrete cylinder compressive strength; FRP thickness; bond length; bond width (i.e. FRP width); and FRP modulus of elasticity. The output parameter was the bond strength.

Zhang et al. [6] applied artificial intelligent techniques for directly predicting the cracking patterns of masonry wallets, subjected to vertical loading. The von Neumann neighborhood model and the Moore neighborhood model of cellular automata (CA) were used to establish the CA numerical model for masonry wallets. Techniques for the analysis of wallets whose bed courses have different angles with the horizontal bottom edges were also introduced. Two criteria were used to match zone similarity between a “base wallet” and any new “unseen” wallets. This zone similarity information was used to predict the cracks in unseen wallets. A back-propagation NN was also used for predicting the cracking pattern of a wallet based on the CA model of the wallet and some data of recorded cracking at zones.

El-Shafie et al. [7] proposed a model based on radial basis function neural networks (RBFNN) for predicting creep in concrete and masonry structures which was compared to a multi-layer perceptron neural network (MLPNN). Accurate prediction of creep was achieved due to the simple architecture and fast training procedure of RBFNN model especially when compared to MLPNN model. The RBFNN model showed good agreement with experimental creep data from brickwork assemblages.

Zhou et al. [8] proposed an artificial intelligence technique for predicting the failure/cracking loads of laterally loaded masonry wall panels based on their corresponding failure/cracking patterns derived from the laboratory experiments. The numerical modes of failure/cracking patterns of experimental wall panels and the corresponding normalized failure/cracking loads were used as the input and output for the NN training. Three types of NN models for predicting the failure/cracking load of the unseen wall panel were achieved by repeatedly training and adjusting to optimize its parameters.

Mathew et al. [9] proposed the use of NN for solving complex nonlinear problems for the analysis of masonry panels under biaxial bending. A Neural Network, trained with the use of a set of data, which was representative of the problem domain, was shown to be successful in solving new problems with reasonable accuracy. The experimental results obtained from the testing of panels were analyzed, and the method that gave good correlation between the theoretical prediction and the experimental result was recommended for other panels of similar properties and boundary conditions. An artificial intelligence based technology, the case-based reasoning (CBR), had been used to solve new problems by adapting solutions to similar problems solved in the past, which were stored in the

case library. A hybrid system was described that utilized the capabilities of both ANNs and CBR.

As a conclusion on the literature review, it is apparent that although Neural Networks have been successfully used in numerous engineering applications in the past, only very few studies have incorporated the use of Neural Networks for the approximation of masonry behavior.

2. Introduction and methodology

Despite the fact that masonry is one of the oldest structural materials and, actually, the main element in monumental structures such as churches, castles, and mosques, our knowledge regarding its mechanical behavior is not as thorough as it should be and many aspects of its behavior remain to be investigated. One reason for this lack of knowledge is the highly anisotropic brittle nature of masonry, which makes complicated, difficult and expensive, the realization of reliable experimental tests under conditions of biaxial stress, and, even more, under conditions of biaxial tension or heterogeneous stress. Taking into account the numerous uncertainties of the problem, a computational model, describing the masonry failure surface in a simple manner should be an efficient tool for the investigation of the behavior of masonry structures. Many analytical criteria for masonry structures have been already proposed [10–15]. Experimental investigations can also be considered as an important support to the aforementioned efforts [16–19].

Researchers have long been aware of the significance of the bed joint angle to the applied load and many experimental tests have been carried out on brick masonry discs to produce indirect tensile stresses on joints inclined at various angles to the vertical compressive load. The highest strength of masonry is observed for cases when the compressive load is perpendicular to the bed joints or in other words when the principal tensile stress at the center of the disc is parallel to the bed joints. In this case failure occurs through bricks and perpendicular joints. The lowest strength is observed when the compressive load is parallel to the bed joints or when the principal tensile stress at the center of the disc is perpendicular to the bed joints. In this case failure occurs along the interface of brick and mortar joint.

Our current work utilized the experimental data reported by Page [16], referring to a total of 102 panels, which have been already used by many other researchers [1,11,20]. Ratios of vertical compressive stress σ_I to horizontal compressive stress σ_{II} of 1, 2, 4, 10 and ∞ (uniaxial σ_I) have been used in conjunction with a bed joint angle θ with respect to σ_I , in directions of 0°, 22.5°, 45°, 67.5° and 90°. A minimum of four tests were performed for each combination of σ_I/σ_{II} and θ .

This data set has been used in the framework a novel methodology, proposed in the present study, which applies Neural Networks in order to approximate the experimental failure curves of a brittle anisotropic material such as masonry. The aim of the study was to introduce an anisotropic (orthotropic) Neural Network – generated 3D failure surface under biaxial stress for masonry for any angle of the joints to the vertical compressive load, as described in detail in the next paragraphs. First, for each angle θ (0°, 22.5°, 45°) of the joints to the vertical compressive load, a Neural Network was trained with the experimental data of Page as inputs (3 NNs in total). Then each one of the three NNs was asked to produce the whole 2D failure curve for each angle as its output, filling also the gaps between the experimental points, thus “enriching” the experimental data with appropriate approximations. Then another bigger, “global” NN was trained using the results of the three NNs as inputs with the angle θ as an input, also. The new NN was then asked to fill also the gaps between the angles

θ , providing the whole 3D failure surface for any angle θ and any ratio of σ_I/σ_{II} .

3. Biaxial testing procedure

Masonry as a composite material consisting of bricks-and-mortar joints is a multi-phase material that depicts distinct brittle and strongly anisotropic nature. This anisotropic nature makes complicated, difficult and expensive, the realization of reliable experimental tests under conditions of biaxial stress, and, even more, under conditions of biaxial tension or heterogeneous stress. In the next paragraphs, the process of both the preparation of test specimens and the tests conducted under biaxial stress state are presented in detail, with special attention to the parameters that affect the results. Furthermore, this detailed presentation will contribute to better understanding the masonry failure as well as to the better understanding of the method (simulation) since the results of the masonry failure are used as inputs to the NN model in the present study.

3.1. Preparation of test specimens

There are only a limited number of experiments on masonry under biaxial stress in the literature; among them are the classical experimental works of Samarasinghe [17], Page [16], and Tassios and Vachliotis [19], the findings of which have been used widely by the research community in the last 30 years for various purposes. A common characteristic of these works is that the final specimens with the correct shape and size are derived by one of the following two ways:

First way: According to Samarasinghe [17], specimens with horizontal and vertical joints (Fig. 1a) are initially constructed with dimensions greater than those of the final specimens. Then a square with the desired dimensions is drawn on the surface of the wall by a pencil at the appropriate orientation to achieve the correct lay-up angle as shown in Fig. 1a.

The lay-up angle is defined as the angle between the direction of the bed joints and one of the edges of the finished test specimen. Therefore, bed joints run at oblique incidence to the edges of the finished saw-cut specimen. Five lay-up angles were selected for biaxial tests of the present study, namely 0° , 22.5° , 45° , 67.5° and 90° .

After a time-span of fourteen days the larger wall panels were cut to the required size and shape by a 'Clipper' saw (Fig. 1b). Specimens in which the lay-up angle was 0° or 90° , were constructed directly without having to be derived from larger ones since they were of correct shape and size. The 'Clipper' saw has the capacity to hold impregnated diamond edge circular blades of varying thickness and diameters.

It is worth noting that circular blades of different thickness must be used on trial walls to find out the most suitable blade which gives a perfectly smooth cut without damaging the bonds of the wall. As the thickness of the blade becomes larger, more force is transmitted to the wall which causes bond failure.

Generally, two days prior to the date of testing, the 'compressive edge' of the panel (the side on which the compressive loads was going to be applied) was capped with 1:1 (cement:sand) mortar.

Second way: According to Page [16] all specimens are constructed directly to their final shape and size as follows: All brickwork is constructed horizontally on a rigid form with bricks glued to a Perspex backing sheet to ensure a constant joint thickness. Panels are made with varying bed joint angles by cutting individual bricks to the required shape before casting.

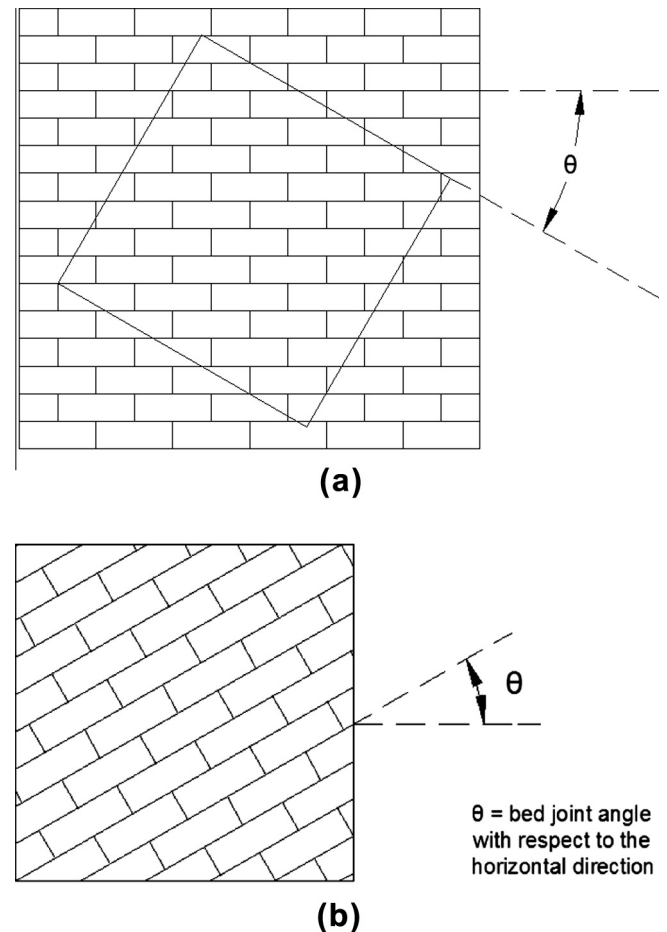


Fig. 1. Preparation of specimens. (a) Wall panel before saw cut. (b) Saw cut specimen.

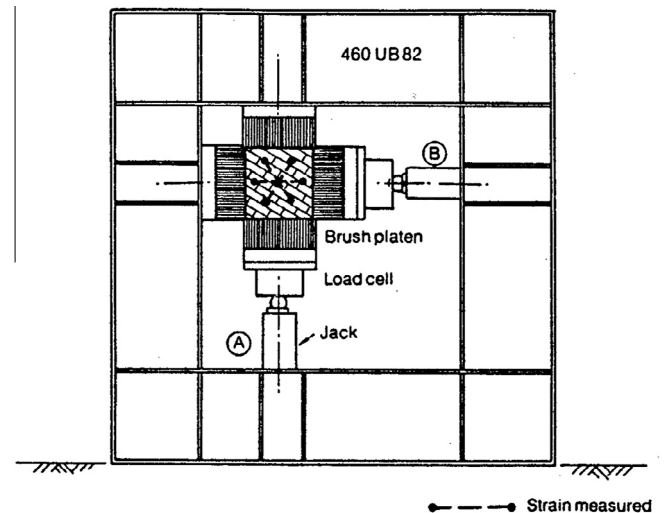


Fig. 2. Biaxial testing rig.

3.2. Testing rig

The biaxial testing rig is shown in Fig. 2. A biaxial stress state is induced in the panel by loading with hydraulic jacks in two orthogonal directions. A constant load ratio is maintained during each test by means of the spreader beam. The load in each direction is monitored by load cells immediately adjacent to the specimen.

4. Biaxial compression tests

A total of 102 panels were tested by Page in 1981 [16]. Ratios of compressive stress σ_I to horizontal compressive stress σ_{II} of 1, 2, 4, 10 and infinity (i.e. uniaxial σ_I) were used in conjunction with bed joint angle θ with respect to the σ_I direction of 0° , 22.5° , 45° , 67.5° and 90° . Fig. 3 shows the saw-cut specimens and stress directions for the three cases $\vartheta = 0.0^\circ$, $\vartheta = 22.5^\circ$ and $\vartheta = 45.0^\circ$. Principal stress ratios of 0 (i.e. uniaxial σ_{II}), 0.1, 0.25, 0.5 and 1 were obtained from the results using the symmetry of the panels and the loading. A minimum of four tests were performed for each combination of σ_I/σ_{II} and θ . The failure envelopes that Page obtained by plotting mean curves for each bed joint angle are shown in Figs. 4–6. These failure envelopes are based on the peak strength values obtained from running bond masonry panel tests in which a uniform loading was applied proportionally. It should be noted that in these figures, and also in all other similar figures of this paper, positive stress denotes compression.

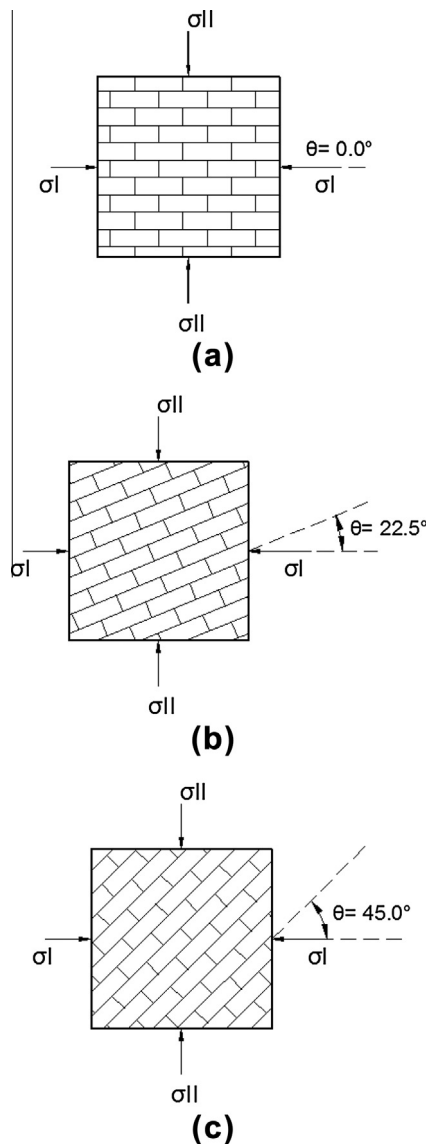


Fig. 3. Saw-cut specimens and stress directions: (a) $\vartheta = 0.0^\circ$, (b) $\vartheta = 22.5^\circ$, (c) $\vartheta = 45.0^\circ$ [16].

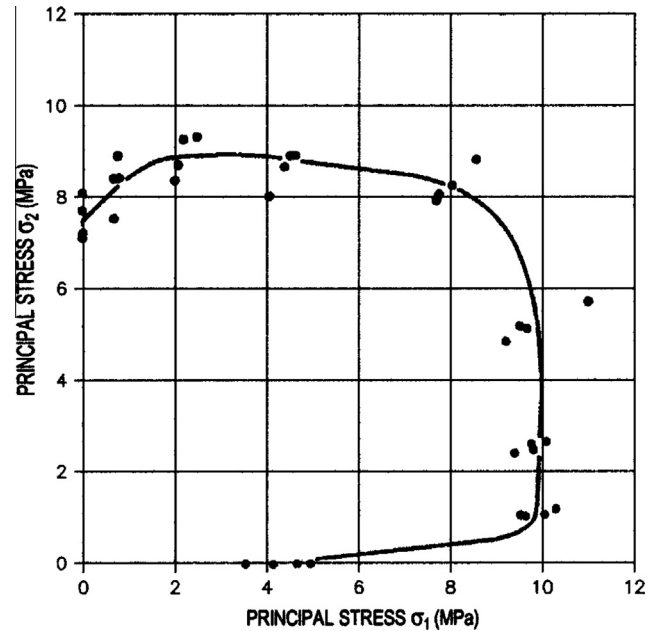


Fig. 4. Failure curve of brick work under biaxial compression ($\vartheta = 0^\circ$) [16].

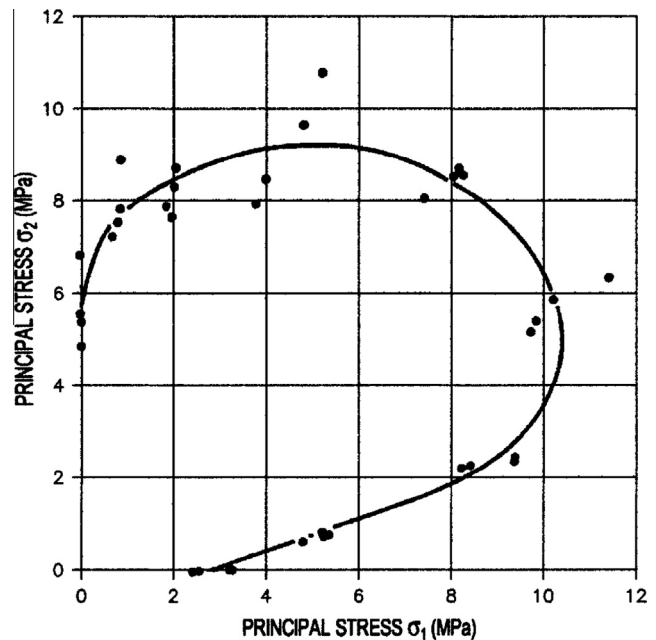


Fig. 5. Failure curve of brick work under biaxial compression ($\vartheta = 22.5^\circ$) [16].

5. Artificial Neural Networks

The development of Artificial Neural Networks (NNs) was initially inspired and motivated by insights into how biological brains – and in particular mammalian brains – function. It was found that mammalian brains learn as connections between neurons are strengthened – the result of electrochemical processes triggered by external or internal stimuli (experiences). As in biological systems, learning involves adjustments to the synaptic connections that exist between the neurons. NNs, like human beings, learn by example.

Although parallels with biological systems are often described, there is still so little known (even at the lowest cell level) about biological systems, that the models being used for artificial neural

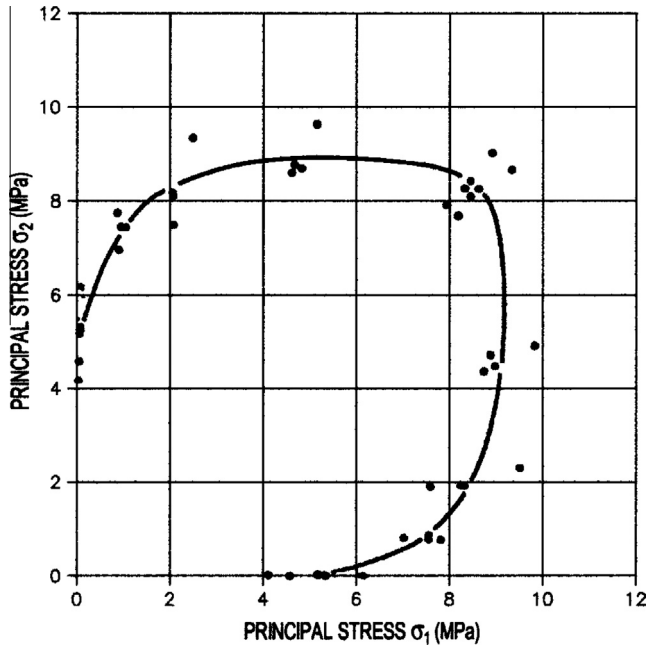


Fig. 6. Failure curve of brick work under biaxial compression ($\phi = 45.0^\circ$) [16].

systems seem to introduce an oversimplification of the ‘biological’ models. The real, biological nervous system is highly complex and includes some features that may seem superfluous based on the understanding of artificial networks.

In the first work on the processing of Neural Networks [21], it was shown theoretically that networks of artificial neurons could implement logical, arithmetic, and symbolic functions. Simplified models of biological neurons were set up, now usually called perceptrons or artificial neurons. These simple models accounted for neural summation, i.e. potentials at the post-synaptic membrane would sum in the cell body. Later models also provided for excitatory and inhibitory synaptic transmission.

Artificial Neural Networks are made up of fully or partially interconnecting artificial neurons (programming constructs that mimic the properties of biological neurons). Artificial NNs may either be used to gain an understanding of biological Neural Networks, or for solving artificial intelligence problems without necessarily creating a model of a real biological system. *Artificial Intelligence* (AI) and *cognitive modeling* try to simulate some properties of Neural Networks. While similar in their techniques, AI has the aim of solving particular tasks, while cognitive modeling aims to build mathematical models of biological neural systems. In the AI field, Artificial Neural Networks have been trained to perform complex functions in various scientific fields and have been applied successfully to identification, classification, simulation, inverse simulation, speech recognition, pattern recognition, image analysis and adaptive control, and also in order to construct software agents (in computer applications) or autonomous robots.

5.1. Neural Networks architecture

In the present study, we use *Back-Propagation Neural Networks* (BPNNs). In this type of NNs, the output values are compared with the correct answer to compute the value of a predefined error-function. By various techniques, the error is then fed back through the network. Using this information, the algorithm adjusts the weights of each connection in order to reduce the value of the error function by some small amount. After repeating this process for a sufficiently large number of training cycles, the network will

usually converge to some state where the error of the calculations is small. In this case, one would say that the network has *learned* a certain target function. As the algorithm’s name implies, the errors propagate backwards from the output nodes to the inner nodes. So technically speaking, back-propagation is used to calculate the gradient of the error of the network with respect to the network’s modifiable weights. To adjust weights properly, one applies a general method for non-linear optimization that is called gradient descent. In order to minimize the error, the derivative of the error function with respect to the network weights is calculated, and the weights are then changed such that the error decreases (thus going downhill on the surface of the error function). For this reason, back-propagation can only be applied on networks with differentiable activation functions. Back-propagation usually allows quick convergence on satisfactory local minima for error in the kind of networks to which it is suited.

A BPNN is a feed-forward, multilayer network of standard structure, i.e. neurons are not connected in the layer but they join the layer neuron with all the neurons of previous and subsequent layers, respectively. A BPNN has a standard structure that can be written in short as

$$N - H_1 - H_2 - \dots - H_{NL-1} - M \quad (1)$$

where N is the number of inputs, H_l is the number of neurons in the l -th hidden layer, NL is the number of layers (including the output layer) and M is the number of output neurons. Fig. 7 depicts an example of a BPNN composed of an input layer with 4 neurons, two hidden layers with 3 neurons each and an output layer with 2 neurons, i.e. a 4-3-3-2 BPNN.

A single node (neuron) of a hidden layer, with a single R -element input vector is shown in Fig. 8, using another kind of notation.

For each node, the individual element inputs $p_1, \dots, p_R$ are multiplied by the weights $w_{1,1}, \dots, w_{1,R}$ and the weighted values are fed to the summing junction. The sum in matrix form is $\mathbf{W} \cdot \mathbf{p}$, the dot product of the (single row) matrix $\mathbf{W} = [w_{1,1}, \dots, w_{1,R}]$ and the vector $\mathbf{p} = [p_1, \dots, p_R]^T$. The neuron has a bias b , which is summed with the weighted inputs to form the net input n which is the argument of the transfer function f :

$$n = w_{1,1}p_1 + w_{1,2}p_2 + \dots + w_{1,R}p_R + b = \mathbf{W} \cdot \mathbf{p} + b \quad (2)$$

In the above case, $\mathbf{p}$ is a column vector ($R \times 1$), $\mathbf{W}$ is a row vector ($1 \times R$) and b is a scalar.

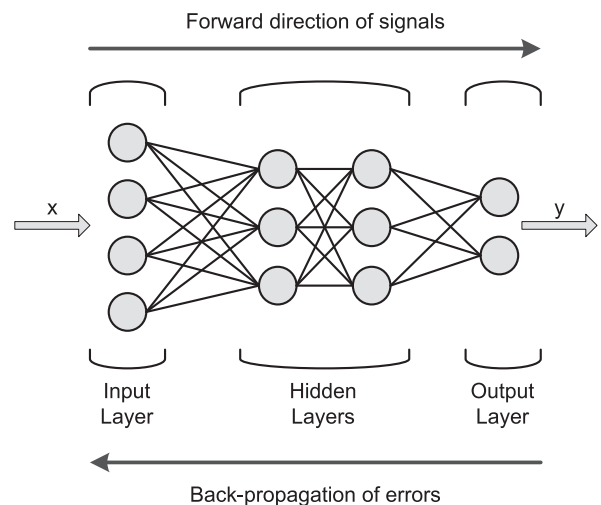


Fig. 7. A three-layer 4-3-3-2 BPNN (input not counted as a layer).

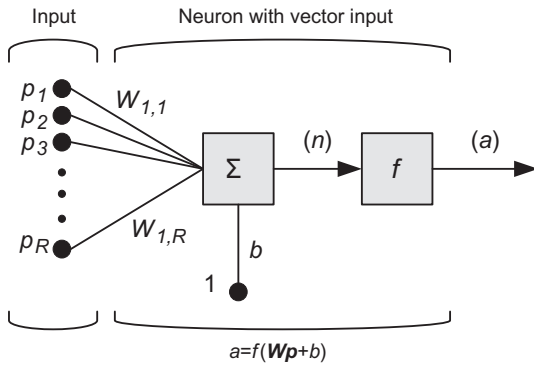


Fig. 8. A neuron with a single R-element input vector.

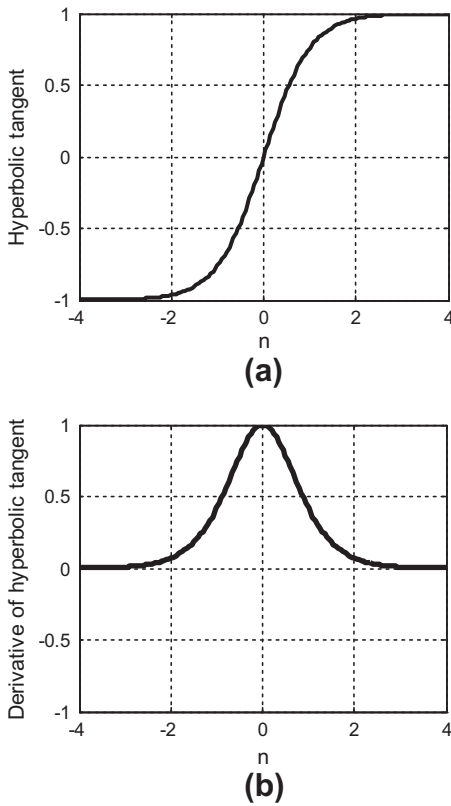


Fig. 9. (a) Hyperbolic tangent function, and (b) its derivative.

5.2. Transfer functions

The choice of the transfer function may strongly influence complexity and performance of Neural Networks. Sigmoidal transfer functions are the most commonly used, while a large number of alternative transfer functions have been also described in the literature [22]. The transfer function used in the present study is the hyperbolic tangent function, the same for all the hidden and the input layer, while the transfer function for the output layer is a linear function. This scheme has been used in all the NNs of the present study. The output of the hyperbolic tangent function and its derivative are given by

$$a = f(n) = \tanh(n) = \frac{e^{2n} - 1}{e^{2n} + 1} = -i \cdot \tan(in) \quad (3)$$

$$f'(n) = 4 \frac{e^{2n}}{(e^{2n} + 1)^2} = 1 - a^2 \quad (4)$$

This function yields output values in the interval $[-1, 1]$, while its derivative yields output values in the interval $[0, 1]$. The graphical representations of the hyperbolic tangent function and its derivative are shown in Fig. 9.

6. Three NN models (for each angle θ)

6.1. NN architecture

We used a Back-Propagation Neural Network with two hidden layers, one input layer and one output layer for all three NNs. The input layer had one node (neuron) which corresponds to the angle φ which defines the ratio σ_{II}/σ_I , while the output layer had also one node which corresponds to the radius r of the point on the failure curve. These two important parameters (φ and r) will be described in detail in the following paragraphs. The two hidden layers had 8 nodes each for the first two cases ($\theta = 0^\circ$ and $\theta = 22.5^\circ$), ending in a 1-8-8-1 BPNN architecture. For the third case ($\theta = 45^\circ$) the two hidden layers had 12 nodes each, ending in a 1-12-12-1 BPNN architecture. These values have been chosen after some trial tests on various network architectures. The reason that the third NN requires more nodes in the hidden layer has probably to do with the number of the input data, which is different in the third case (10 points as opposed to 9 points for the first two cases, as shown in Figs. 10–12).

The input and output values are normalized before the NN training and the inverse normalization is performed in order to take the NN results for other data afterwards.

6.2. Preparation of the NN input data

The experimental data of Page [16] have been used as inputs for the first three NN models of the present study. Figs. 10–12 show the original data (in a normalized form) together with the average values (as squares) that attempt to fit these data for each bed joint angle.

It should be noted that the data for $\theta = 45^\circ$ are symmetric to the 45° line ($\sigma_I = \sigma_{II}$), due to the nature of the loading, while the cases $\theta = 67.5^\circ$ and $\theta = 90^\circ$ are “equivalent” to the cases $\theta = 22.5^\circ$

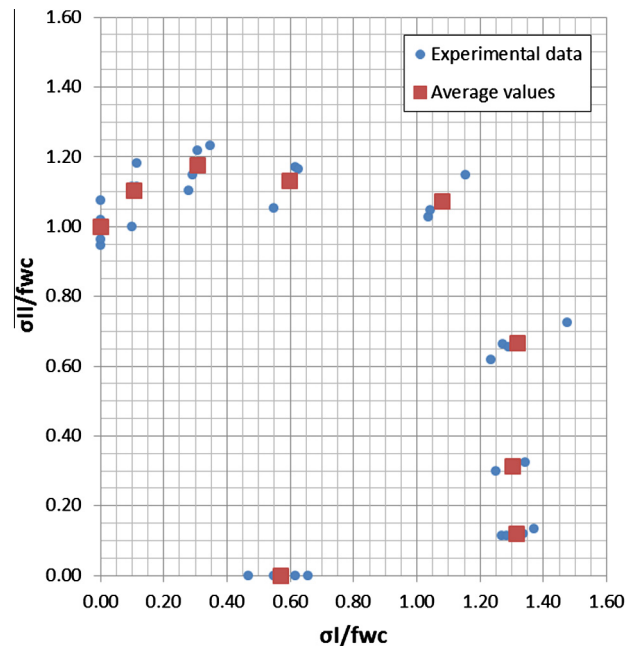


Fig. 10. Normalized experimental data and average values for the $\theta = 0^\circ$ case.

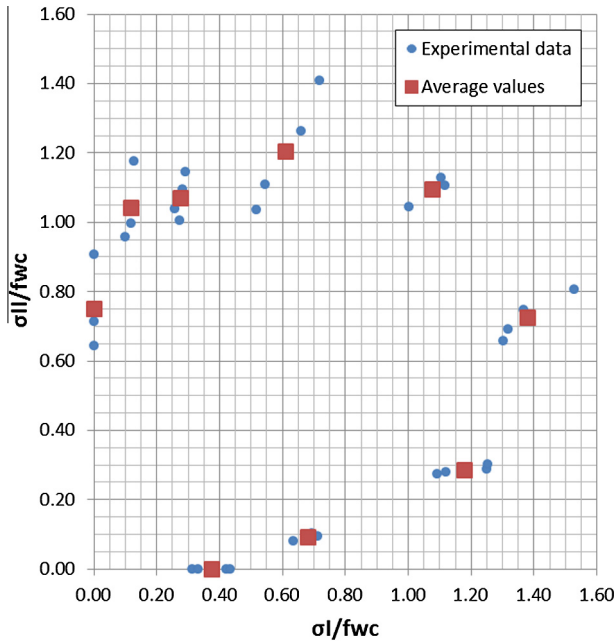


Fig. 11. Normalized experimental data and average values for the $\theta = 22.5^\circ$ case.

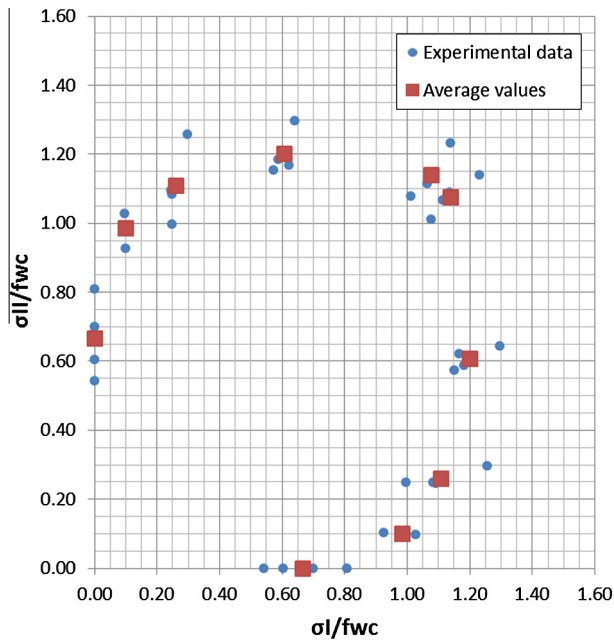


Fig. 12. Normalized experimental data and average values for the $\theta = 45^\circ$ case.

and $\theta = 0^\circ$ (“mirrored” results), respectively and should not be examined separately. Table 1 shows the analytical data for the $\theta = 0^\circ$ case.

The first two columns of the table contain the original experimental data, namely the failure principal stresses σ_I and σ_{II} in MPa. For every loading case, four tests have been conducted. The next two columns contain the same data in a dimensionless form where the stresses have been divided with the stress f_{wc} which is the masonry strength for the case $\sigma_I = 0$ (last four rows of the table). The value of f_{wc} has been calculated as the average of the four values (highlighted in bold in Table 1) as 7.56 MPa.

The next two columns are the averages of the four tests for each loading case.

For the data to be suitable for the Neural Network training, a conversion to polar coordinates (r, φ) is needed, where the radius r is given by

$$r = \sqrt{\sigma_{I,aver}^2 + \sigma_{II,aver}^2} \quad (5)$$

where $\sigma_{I,aver}$ and $\sigma_{II,aver}$ are the average stresses for each loading case (columns 5, 6). The polar angle φ (column 7) is given by

$$\varphi = \text{Arc tan} \left(\frac{\sigma_{II}}{\sigma_I} \right) \quad (6)$$

The polar radius r is given in the last column (column 8) of Table 1. For each of the three angles θ ($\theta = 0^\circ, 22.5^\circ, 45^\circ$), a Neural Network is trained with the angle φ as its input and the radius r as its output. Only the average values of each test (denoted as “square” points in Figs. 10–12) are used as training data for the Neural Network.

Figs. 10–12 and Tables 1–3 show the corresponding data for the other two cases ($\theta = 22.5^\circ$ and $\theta = 45^\circ$).

6.3. Approximation results of the three NNs

The three NNs were trained with the input and output data of Tables 1–3 (last two columns) and then each NN was asked to produce the full curves for each bed joint angle, for a set of 100 segments (101 points). The results are shown in Figs. 13–15.

In Figs. 13–15, the black dots denote the input data, i.e. the data corresponding to the last two columns of Tables 1–3. The blue curve denotes the NN prediction of the fitting curve.

It can be seen that the NN managed to fit all the training data with excellent accuracy, while the approximation between the training data points appears to be adequate. An important characteristic is that for the third case ($\theta = 45^\circ$) the NN managed to produce results that are symmetric (with good accuracy) to the line of 45° ($\sigma_I = \sigma_{II}$), an expected outcome due to the symmetry that masonry exhibits with respect to the 45° axis.

7. “Global” Neural Network model

In the previous part of the study, three NN Models were trained, each NN for an angle θ ($0^\circ, 22.5^\circ$ and 45°). The final purpose of the study is to add also the angle θ as a parameter of the problem, thus creating a model that will be able to predict the failure curve not only for the predefined angles, but for any angle θ .

In order to achieve this, after the three NNs were trained and they had provided their results, another bigger, “Global” NN was trained which took the results of the three NNs as inputs with the angle θ as an input, also. The new NN was then asked to fill also the gaps between the angles θ , providing the whole 3D failure surface for any angle θ and any ratio of σ_{II}/σ_I .

The Global NN is also a BPNN with an architecture of 9 nodes per hidden layer (2-9-9-1). The two inputs are the angles φ ($0^\circ - 90^\circ$) and θ ($0^\circ - 45^\circ$), while the output is the radius r . The transfer function of the global NN is the hyperbolic tangent function that was used also in the first three NNs. For the global NN, we use the results of the other three NNs as training patterns. This means that we have $101 \times 3 = 303$ training patterns, as we use 101 points for every angle θ ($0^\circ, 22.5^\circ$ and 45°).

Table 1Failure of brickwork under biaxial compression, $\theta = 0^\circ$ [16] and relevant calculations.

σ_I (MPa)	σ_{II} (MPa)	σ_{II}/f_{wc}	σ_{III}/f_{wc}	Average σ_{II}/f_{wc}	Average σ_{III}/f_{wc}	Angle $\varphi = \text{Atan}(\sigma_{II}/\sigma_I)$	Radius r
3.53	0.00	0.47	0.00				
4.14	0.00	0.55	0.00				
4.66	0.00	0.62	0.00				
4.96	0.00	0.66	0.00	0.57	0.00	0.0000	0.5717
9.59	0.85	1.27	0.11				
9.70	0.87	1.28	0.12				
10.11	0.90	1.34	0.12				
10.37	1.01	1.37	0.13	1.32	0.12	0.0909	1.3210
9.46	2.26	1.25	0.30				
9.85	2.43	1.30	0.32				
9.92	2.30	1.31	0.30				
10.15	2.45	1.34	0.32	1.30	0.31	0.2353	1.3398
9.33	4.67	1.23	0.62				
9.62	5.01	1.27	0.66				
9.74	4.95	1.29	0.65				
11.15	5.48	1.48	0.72	1.32	0.66	0.4672	1.4761
7.84	7.76	1.04	1.03				
7.89	7.91	1.04	1.05				
8.20	8.10	1.09	1.07				
8.72	8.67	1.15	1.15	1.08	1.07	0.7820	1.5222
4.16	7.95	0.55	1.05				
4.55	8.57	0.60	1.13				
4.66	8.84	0.62	1.17				
4.74	8.81	0.63	1.17	0.60	1.13	1.0836	1.2792
2.12	8.34	0.28	1.10				
2.21	8.67	0.29	1.15				
2.32	9.21	0.31	1.22				
2.63	9.32	0.35	1.23	0.31	1.18	1.3156	1.2148
0.76	7.55	0.10	1.00				
0.76	8.42	0.10	1.11				
0.87	8.42	0.11	1.11				
0.87	8.93	0.11	1.18	0.11	1.10	1.4737	1.1074
0.00	7.15	0.00	0.95				
0.00	7.27	0.00	0.96				
0.00	7.69	0.00	1.02				
0.00	8.12	0.00	1.07	0.00	1.00	1.5708	1.0000

Table 2Failure of brickwork under biaxial compression, $\theta = 22.5^\circ$ [16] and relevant calculations.

σ_I (MPa)	σ_{II} (MPa)	σ_{II}/f_{wc}	σ_{III}/f_{wc}	Average σ_{II}/f_{wc}	Average σ_{III}/f_{wc}	Angle $\varphi = \text{Atan}(\sigma_{II}/\sigma_I)$	Radius r
2.37	0.00	0.31	0.00				
2.50	0.00	0.33	0.00				
3.19	0.00	0.42	0.00				
3.28	0.00	0.43	0.00	0.38	0.00	0.0000	0.3752
4.79	0.60	0.63	0.08				
5.24	0.78	0.69	0.10				
5.22	0.71	0.69	0.09				
5.38	0.71	0.71	0.09	0.68	0.09	0.1352	0.6890
8.26	2.08	1.09	0.27				
8.47	2.12	1.12	0.28				
9.44	2.18	1.25	0.29				
9.47	2.27	1.25	0.30	1.18	0.29	0.2382	1.2132
9.84	4.96	1.30	0.66				
9.95	5.22	1.32	0.69				
10.33	5.65	1.37	0.75				
11.56	6.10	1.53	0.81	1.38	0.73	0.4843	1.5581
7.57	7.89	1.00	1.04				
8.22	8.31	1.09	1.10				
8.35	8.53	1.10	1.13				
8.46	8.35	1.12	1.10	1.08	1.09	0.7927	1.5359
3.91	7.83	0.52	1.04				
4.12	8.37	0.54	1.11				
4.98	9.54	0.66	1.26				
5.43	10.65	0.72	1.41	0.61	1.20	1.1018	1.3493

Table 2 (continued)

σ_I (MPa)	σ_{II} (MPa)	σ_{II}/f_{wc}	σ_{II}/f_{wc}	Average σ_{II}/f_{wc}	Average σ_{II}/f_{wc}	Angle $\varphi = \text{Atan}(\sigma_{II}/\sigma_I)$	Radius r
2.06	7.59	0.27	1.00				
1.95	7.84	0.26	1.04				
2.15	8.27	0.28	1.09				
2.20	8.65	0.29	1.14	0.28	1.07	1.3178	1.1053
0.76	7.23	0.10	0.96				
0.89	7.53	0.12	1.00				
0.93	7.80	0.12	1.03				
0.98	8.89	0.13	1.18	0.12	1.04	1.4580	1.0467
0.00	4.87	0.00	0.64				
0.00	5.39	0.00	0.71				
0.00	5.59	0.00	0.74				
0.00	6.85	0.00	0.91	0.00	0.75	1.5708	0.7508

Table 3

Failure of brickwork under biaxial compression, $\theta = 45^\circ$ [16] and relevant calculations.

σ_I (MPa)	σ_{II} (MPa)	σ_{II}/f_{wc}	σ_{II}/f_{wc}	Average σ_{II}/f_{wc}	Average σ_{II}/f_{wc}	Angle $\varphi = \text{Atan}(\sigma_{II}/\sigma_I)$	Radius r
4.09	0.00	0.54	0.00				
4.56	0.00	0.60	0.00				
5.14	0.00	0.68	0.00				
5.29	0.00	0.70	0.00				
6.11	0.00	0.81	0.00	0.67	0.00	0.0000	0.6666
6.99	0.77	0.93	0.10				
7.52	0.72	0.99	0.10				
7.52	0.80	0.99	0.11				
7.76	0.74	1.03	0.10	0.99	0.10	0.1012	0.9905
7.54	1.88	1.00	0.25				
8.19	1.88	1.08	0.25				
8.27	1.85	1.09	0.24				
9.50	2.24	1.26	0.30	1.11	0.26	0.2301	1.1379
8.71	4.32	1.15	0.57				
8.83	4.70	1.17	0.62				
8.94	4.45	1.18	0.59				
9.80	4.86	1.30	0.64	1.20	0.61	0.4677	1.3446
8.14	7.64	1.08	1.01				
8.41	8.05	1.11	1.07				
8.59	8.22	1.14	1.09				
9.32	8.61	1.23	1.14	1.14	1.08	0.7565	1.5675
7.64	8.14	1.01	1.08				
8.05	8.41	1.07	1.11				
8.22	8.59	1.09	1.14				
8.61	9.32	1.14	1.23	1.08	1.14	0.8143	1.5675
4.32	8.71	0.57	1.15				
4.70	8.83	0.62	1.17				
4.45	8.94	0.59	1.18				
4.86	9.80	0.64	1.30	0.61	1.20	1.1031	1.3446
1.88	7.54	0.25	1.00				
1.88	8.19	0.25	1.08				
1.85	8.27	0.24	1.09				
2.24	9.50	0.30	1.26	0.26	1.11	1.3407	1.1379
0.77	6.99	0.10	0.93				
0.72	7.52	0.10	0.99				
0.80	7.52	0.11	0.99				
0.74	7.76	0.10	1.03	0.10	0.99	1.4696	0.9905
0.00	4.09	0.00	0.54				
0.00	4.56	0.00	0.60				
0.00	5.14	0.00	0.68				
0.00	5.29	0.00	0.70				
0.00	6.11	0.00	0.81	0.00	0.67	1.5708	0.6666

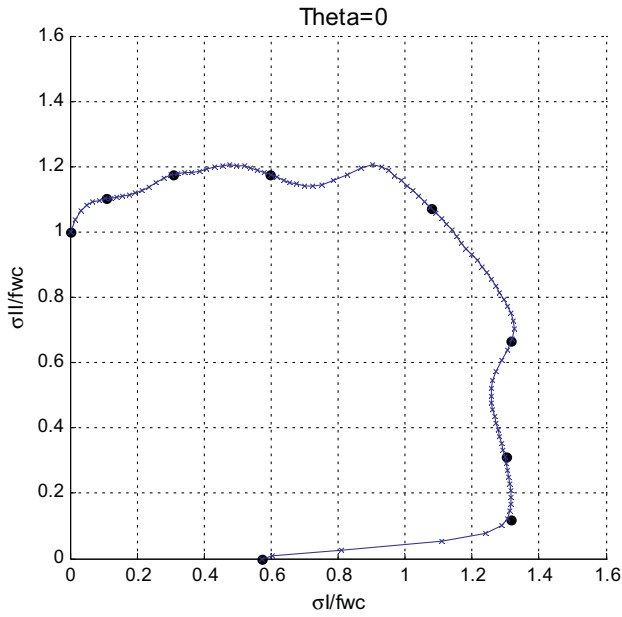


Fig. 13. NN approximation result for the case $\theta = 0^\circ$.

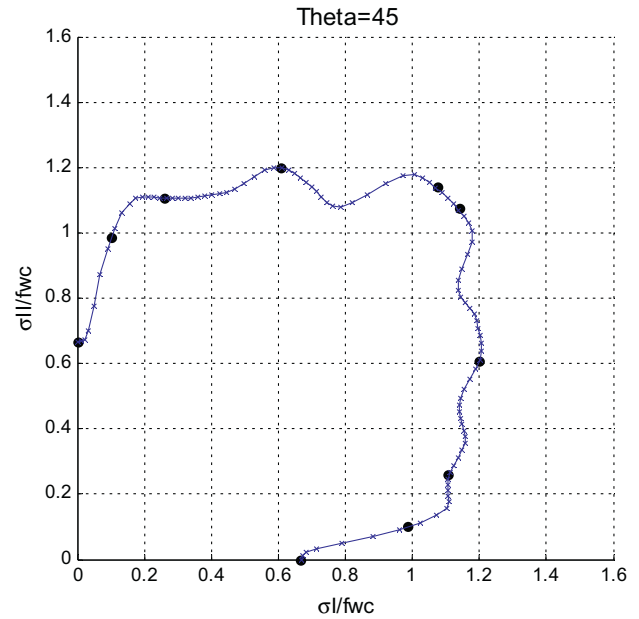


Fig. 15. NN approximation result for the case $\theta = 45^\circ$.

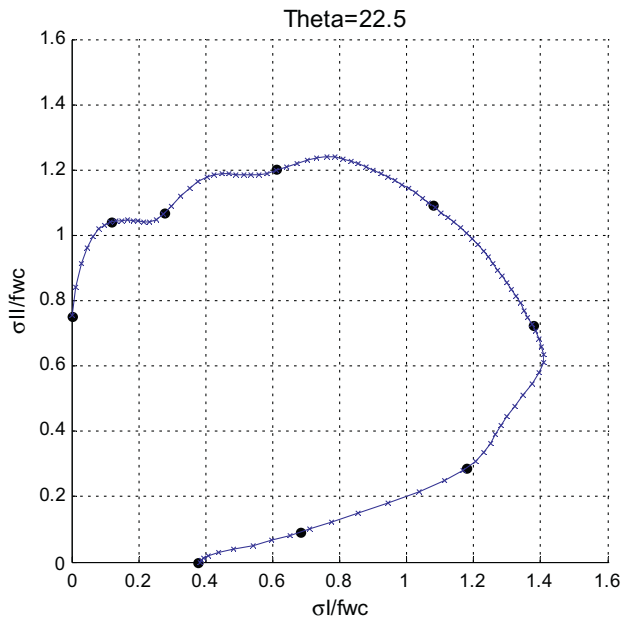


Fig. 14. NN approximation result for the case $\theta = 22.5^\circ$.

7.1. Global NN results

The global NN was trained and then it was asked to produce the whole 3D failure surface. The NN was asked to produce points where the angle φ was divided again in 100 segments (101 points, each segment is equivalent to 0.45°), while angle θ is divided in 64 segments (65 points, each segment is equivalent to 0.703125°). Figs. 16–19 show the result of the NN approximation in 3D. The red points (dots) denote the initial training set of the first three NNs, i.e. the average values gathered from the experimental results and used for the

training of the initial NN. All four figures show the same 3D failure surface, from different viewing angles, for better comprehension.

Then the NN was asked to produce the 2D failure curves for five cases of angle θ (0° , 11.25° , 22.5° , 33.75° and 45°). The results are shown in Figs. 20–24, where for the three cases 0° , 22.5° and 45° there are available experimental data (training sets of the first three NNs), that are denoted with red points (dots).

8. Comparison with other criteria – Discussion

In order to more efficiently validate the obtained results, they are compared to the main analytical criteria used to define the masonry failure surface, presented below:

• Von Mises Failure Criterion

$$\bar{\sigma}_x^2 + \bar{\sigma}_y^2 + 3\bar{\tau}^2 - \bar{\sigma}_x\bar{\sigma}_y = 1 \quad (7)$$

Based upon the Von Mises failure criterion, many modified criteria have been proposed to define the failure of masonry, whereas in the case of biaxial compressive stress the original Von Mises criterion has been used [13,14,7,23–25]. Extensive and in-depth state-of-the-art report on the modified Von Mises Criteria used for masonry can be found in [26,27].

• Second-order polynomial

$$17.15\bar{\sigma}_x + 74.57\bar{\sigma}_y + 32.71\bar{\sigma}_x^2 + 75.34\bar{\sigma}_y^2 + 356.74\bar{\tau}^2 - 25.91\bar{\sigma}_x\bar{\sigma}_y = 1 \quad (8)$$

This criterion comprises a simplified version of the cubic tensor polynomial, which will be presented afterwards. It applies to anisotropic materials and is commonly used to define the failure of masonry materials [20,12].

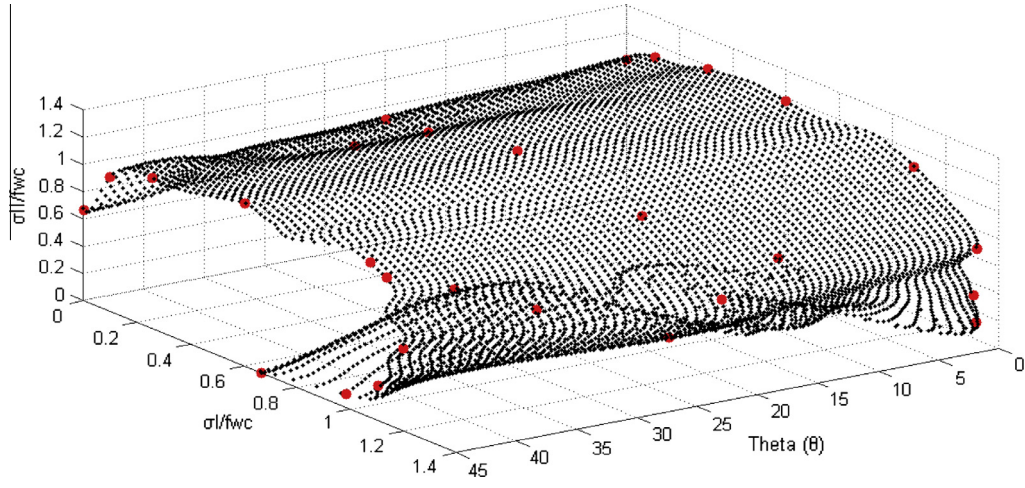


Fig. 16. Global NN approximation result of the 3D failure surface (a).

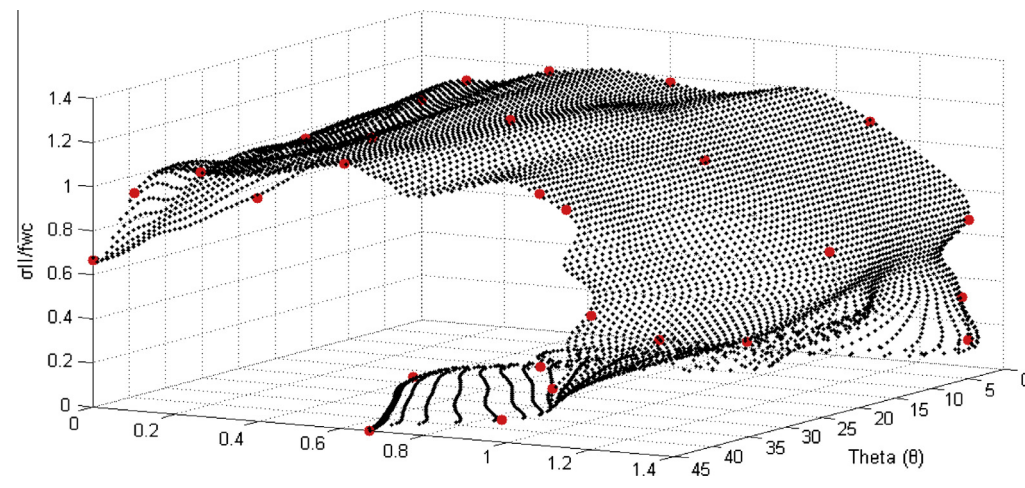


Fig. 17. Global NN approximation result of the 3D failure surface (b).

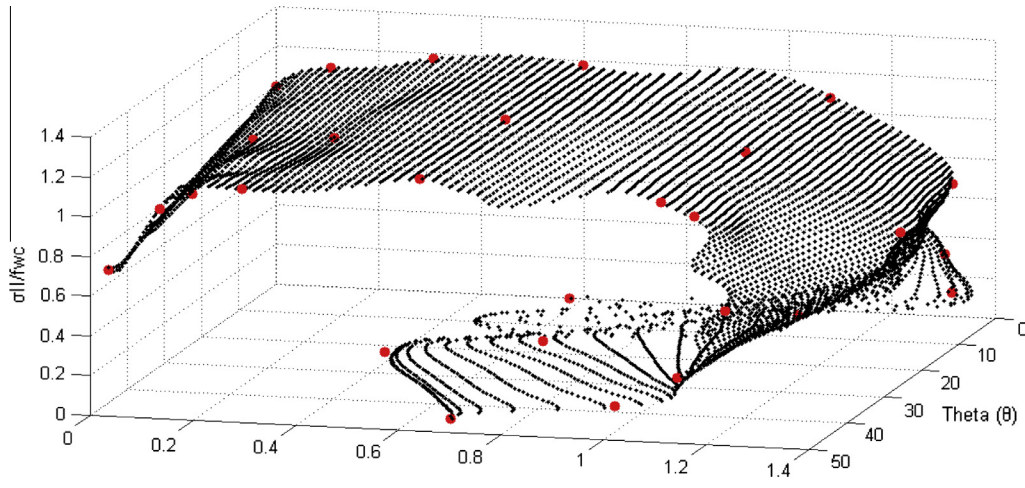


Fig. 18. Global NN approximation result of the 3D failure surface (c).

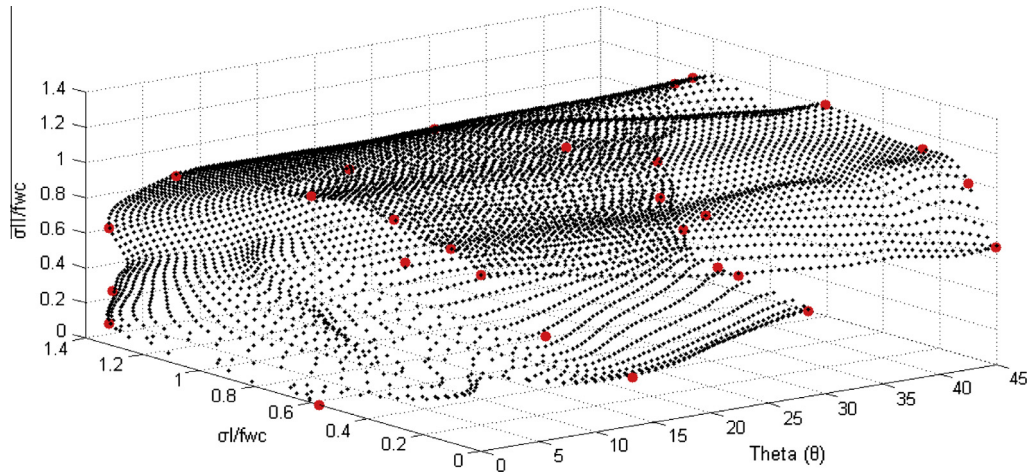


Fig. 19. Global NN approximation result of the 3D failure surface (d).

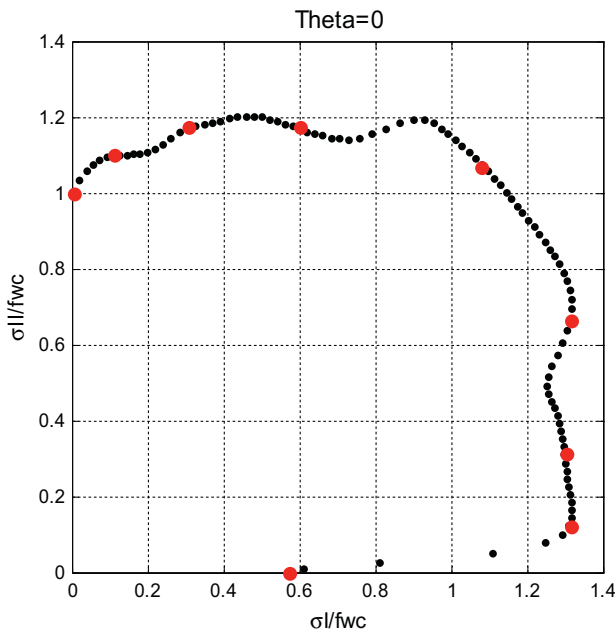


Fig. 20. Global NN approximation for the $\theta = 0^\circ$ case and corresponding training points (red dots). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

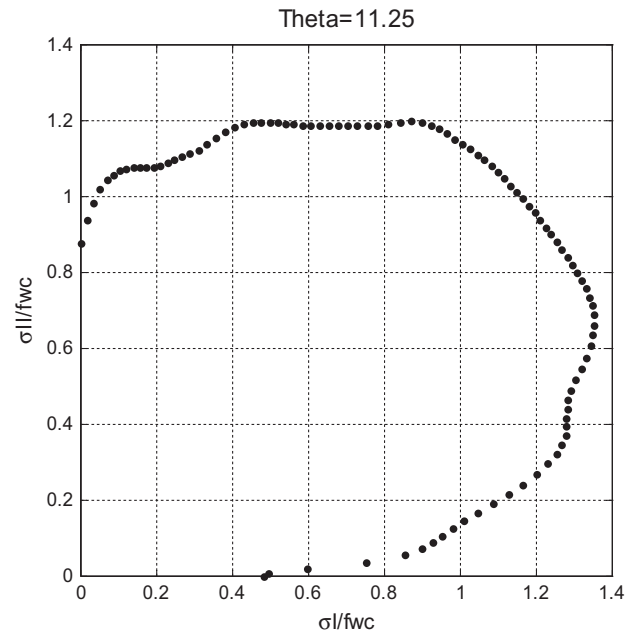


Fig. 21. Global NN approximation for the $\theta = 11.25^\circ$ (no training points available).

- Third-order polynomial or cubic tensor polynomial

$$\left[\begin{aligned} &17.15\bar{\sigma}_x + 74.57\bar{\sigma}_y + 32.71\bar{\sigma}_x^2 + 75.34\bar{\sigma}_y^2 + 356.74\bar{\tau}^2 + \\ &-17.12\bar{\sigma}_x\bar{\sigma}_y + 4.13\bar{\sigma}_x^2\bar{\sigma}_y + 1.35\bar{\sigma}_x\bar{\sigma}_y^2 + 122.46\bar{\sigma}_x\bar{\tau}^2 + \\ &+202.20\bar{\sigma}_y\bar{\tau}^2 = 1 \end{aligned} \right] \quad (9)$$

This criterion comprises the generalization of the aforementioned failure criterion and has been applied to anisotropic composite materials [28–30] including masonry materials [20,15].

All three failure criteria assume plane stress state and are expressed in terms of non-dimensional normal stresses regarding the uniaxial compressive strength perpendicular to the bed joints of masonry $f_{wc}^{90^\circ}$. Namely the non-dimensional normal stresses are defined as:

$$\bar{\sigma}_x = \frac{\sigma_x}{f_{wc}^{90^\circ}} \quad (10)$$

$$\bar{\sigma}_y = \frac{\sigma_y}{f_{wc}^{90^\circ}} \quad (11)$$

$$\bar{\tau} = \frac{\tau}{f_{wc}^{90^\circ}} \quad (12)$$

Figs. 25–29 show the comparison between the results obtained using the proposed NN methodology in relation to the available analytical failure criteria, which were presented above, and the experimental results obtained by Page [16]. These figures show that the proposed NN approximates the experimental results adequately, while (according to the analytical criteria of failure) demonstrate the fact that, the greater the degree of the polynomial function, the better the approximation of the experimental results, as the range of possible shapes increases. It is worth mentioning

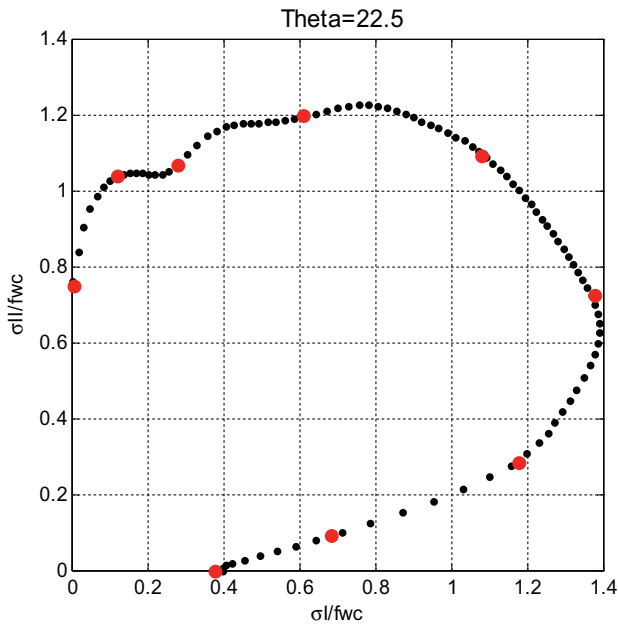


Fig. 22. Global NN approximation for the $\theta = 22.5^\circ$ case and corresponding training points (red dots). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

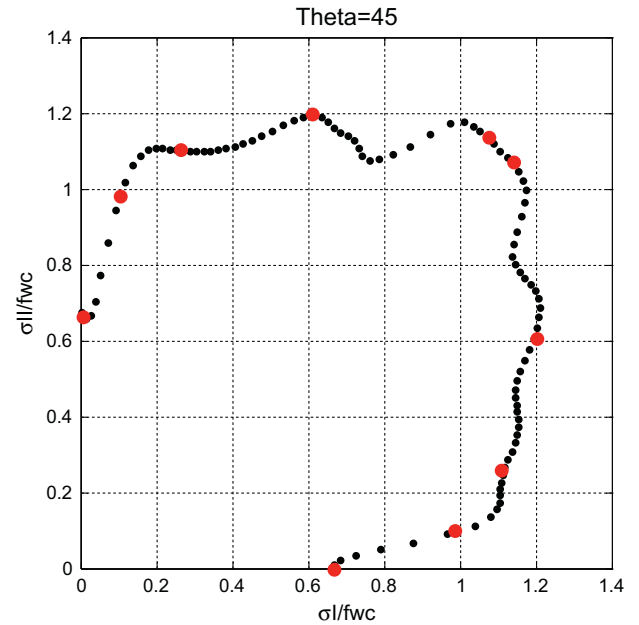


Fig. 24. Global NN approximation for the $\theta = 45^\circ$ case and corresponding training points (red dots). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

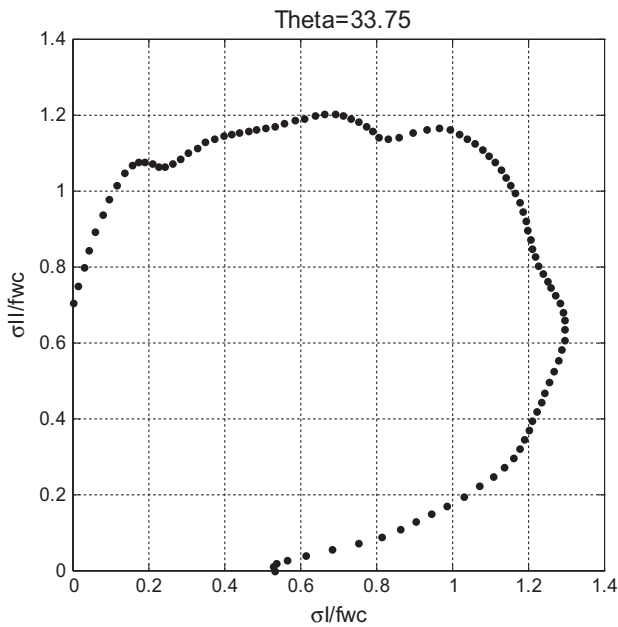


Fig. 23. Global NN approximation for the $\theta = 33.75^\circ$ (no training points available).

the case of the sphere (2nd degree polynomial function), which enables to simulate the experimental results which are always on the sphere's single surface, while the ellipsoid (3rd degree polynomial function) can simulate many experimental results, which are on the surfaces described by it.

It should also be noted that the proposed NN confirms the experimental findings indicating that the failure surface of the material may change the Gaussian Curvature from positive to negative. This has been observed both for isotropic materials such as those described in the classical work of Kupfer, Hilsdorf and Rusch for concrete [31] where the change of curvature is observed for the

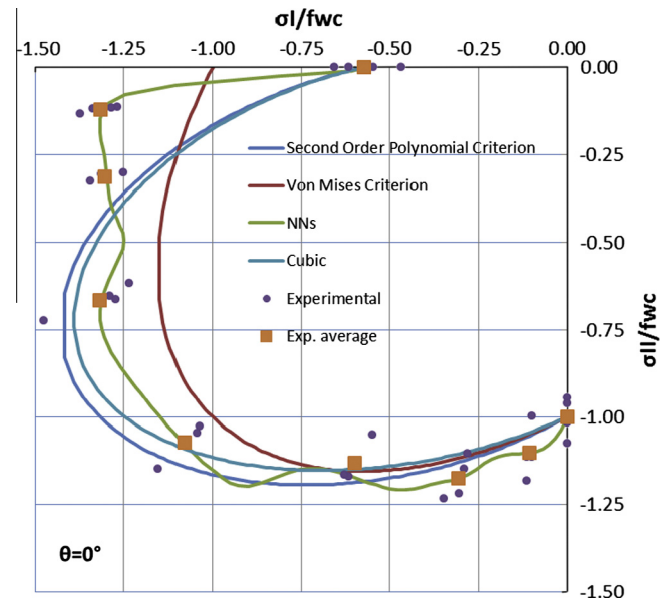


Fig. 25. Comparison of results for $\theta = 0.00^\circ$.

case of heterosemous stress, as well as for the case of anisotropic materials such as the classic case of the masonry that has been thoroughly studied by Page (the experimental results used in this work have been used for the development of the proposed Neural Network). In contrast to the NN, which is able to simulate such a failure surface as a closed surface, the analytical failure criteria, which have been proposed to ensure the closed surface, must always assume a positive Gaussian Curvature [32]. This could be achieved by selecting a fourth-order tensor polynomial, which would unfortunately result in a large number of unknown factors/coefficients, the determination of which is impossible.

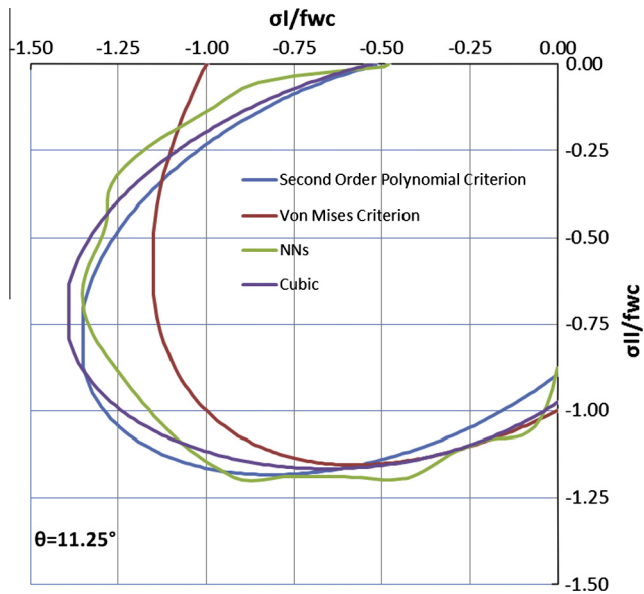


Fig. 26. Comparison of results for $\theta = 11.25^\circ$ (no experimental points available).

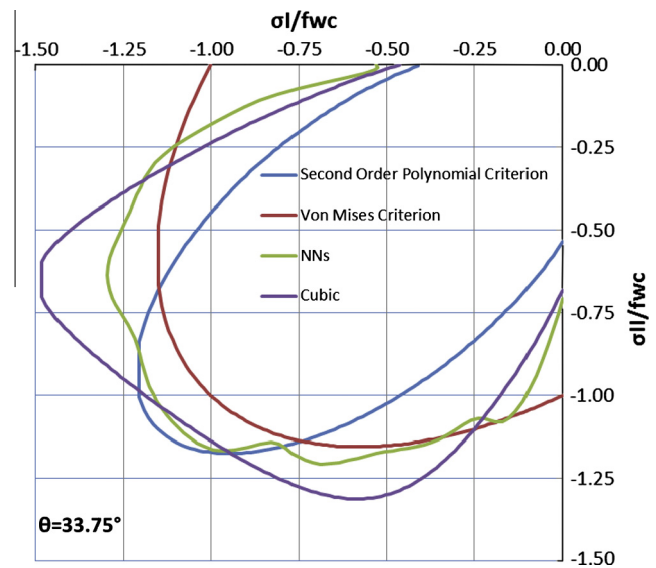


Fig. 28. Comparison of results for $\theta = 33.75^\circ$ (no training points available).

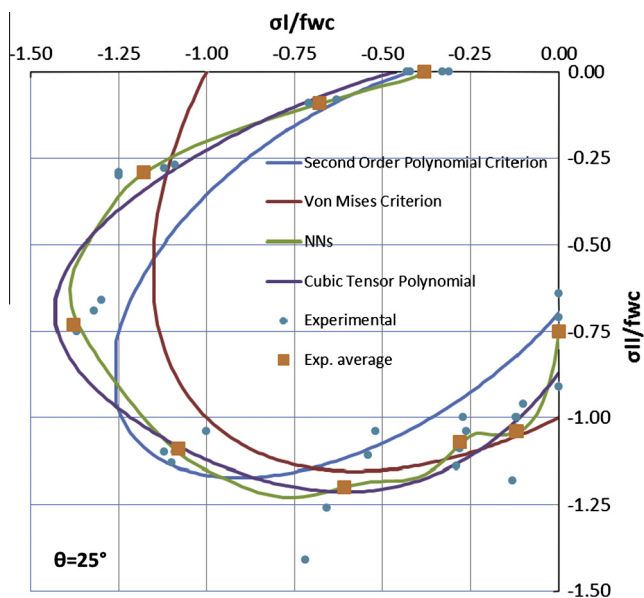


Fig. 27. Comparison of results for $\theta = 25.00^\circ$.

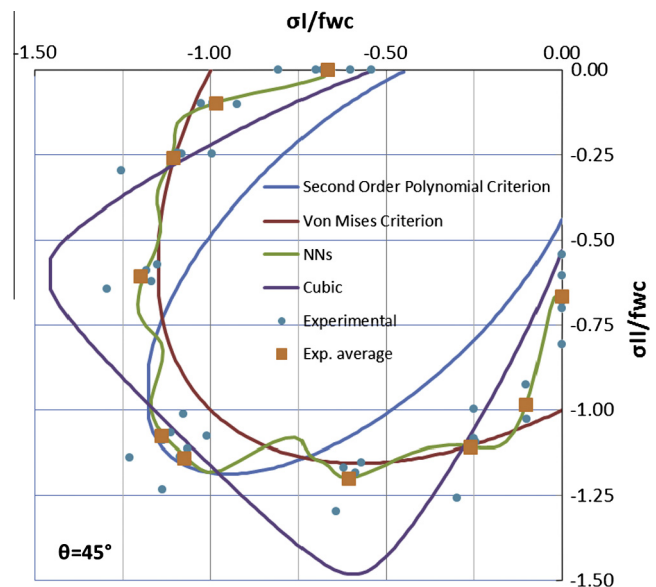


Fig. 29. Comparison of results for $\theta = 45.00^\circ$.

Based on the results of Fig. 6 that referred to the angle θ equal to 45° , the ability of the developed NN to ensure the symmetry of the derived failure curve is clearly depicted. This achievement fulfills the physical requirements that all the materials, both isotropic and anisotropic, have failure curves symmetric to the line of 45° .

Finally, the proposed Neural Network elucidates, for the first time, surface failure areas which have not been explored experimentally. Additionally, it proposes areas to be studied experimentally thus confirming (or not) the validity of the results obtained.

9. Conclusions

In the present study, results on the NN prediction of the failure of anisotropic brittle materials such as masonry are presented. In particular, Neural Networks are used in order to

approximate the experimental results for masonry failure. First, for each angle θ a NN is trained with the experimental data as inputs and then each NN is asked to produce the whole failure curve for each angle as its output, filling also the gaps between the experimental points. Then, another “global” NN is trained which takes the results of the three original NNs as inputs with the angle θ as an input, also. The new NN is then asked to fill also the gaps between the angles θ , providing the whole failure 3D surface for any angle θ . The results show the great potential of using NNs for the approximation of the masonry failure under biaxial compressive stress.

The NNs showed great performance in fitting the experimental input data, while they managed to fit all the training data with very good accuracy, also producing results that are symmetric (with good accuracy) to the line of 45° ($\sigma_I = \sigma_{II}$) for the third case ($\theta = 45^\circ$). This is in very good agreement to the characteristic sym-

metry of masonry. The results show the great potential of using NN for the approximation of the masonry failure under biaxial compressive stress.

Being aware that masonry is a multi-phase material that depicts distinct brittle and anisotropic nature with wide scatter in the values of its mechanical characteristics, the following conclusions can be drawn from this investigation:

- The three first NNs manage to fit the average values of the experimental data with great accuracy. The error for the training data itself is very small, while the NN prediction for data that do not belong to the training data set appears to be also very good, providing smooth curves.
- The curves that are generated by the NNs are continuous and smooth. It should be noted that the NN curves are not necessarily convex, as the curves of the other criteria.
- For the case $\theta = 45^\circ$, the NN manages to provide results that are approximately symmetric to the line $\sigma_I = \sigma_{II}$.
- The “Global” NN manages to give us valuable information about angles that have not been investigated until now. The fit of the experimental data is excellent, while the NN prediction for other points appears to be also very good, providing smooth surfaces in 3D.
- The reliability of the global NN is in general high. In this paper we proposed a computational procedure for approximating failure curves in 3D, rather than giving the final results for this kind of problems. If more dense data were provided to the NN, then it could give even better results. In any case, the quality and quantity of the experimental data is very important in such problems.

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